

# Prediction of unobserved SARS-CoV-2 Infection

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## COVID-19 Dashboard by the Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (JHU)



Last Updated at (M/D/YYYY)

4/21/2022, 4:20 PM

Total Cases

507,528,166

Total Deaths

6,210,798

Total Vaccine Doses Administered

11,213,601,728

Cases | Deaths by

Country/Region/Sovereignty

## Korea, South

28-Day: 5,851,209 | 7,765

Totals: 16,674,045 | 21,667

## Germany

28-Day: 4,627,394 | 6,308

Totals: 23,844,536 | 133,632

## France

28-Day: 3,478,927 | 3,289

Totals: 28,266,009 | 145,836

## Vietnam

28-Day: 2,022,839 | 907

Totals: 10,502,590 | 42,982

## Italy

28-Day: 1,787,992 | 3,844

Totals: 15,934,437 | 162,264

## Australia

28-Day: 1,449,802 | 1,074

Totals: 5,564,147 | 6,893

## United Kingdom

28-Day: 1,391,605 | 8,300

Totals: 22,081,798 | 173,659

## Japan

28-Day: 1,295,286 | 1,780

Totals: 7,531,317 | 29,205

## US

28-Day: 950,971 | 14,737

Totals: 80,840,972 | 990,563

## Thailand

28-Day: 682,274 | 2,895

28-Day Cases

31,209,754

28-Day Deaths

103,248

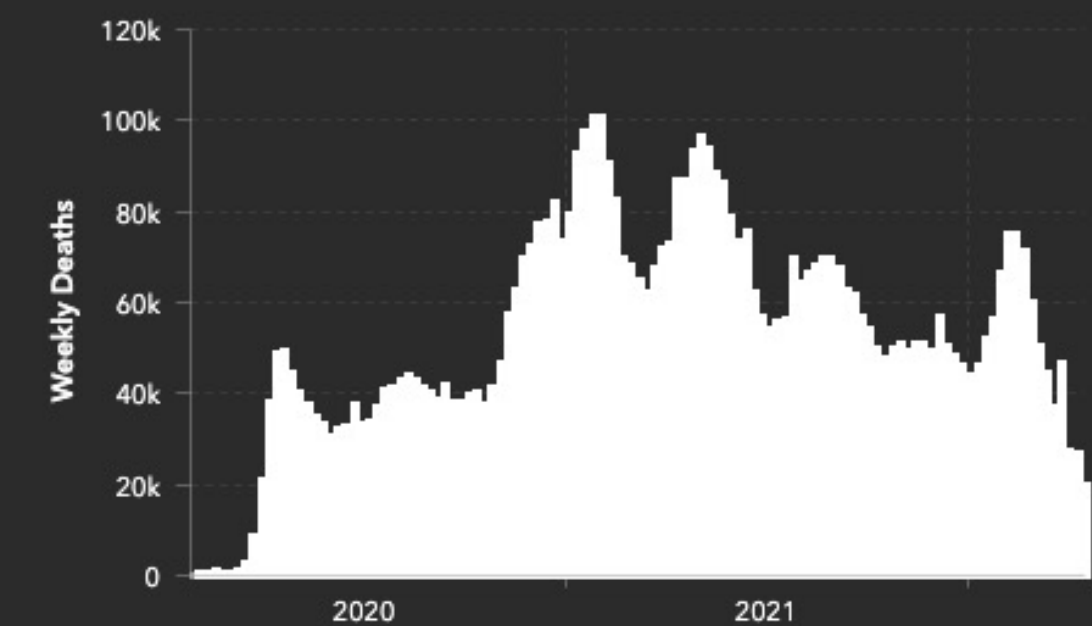
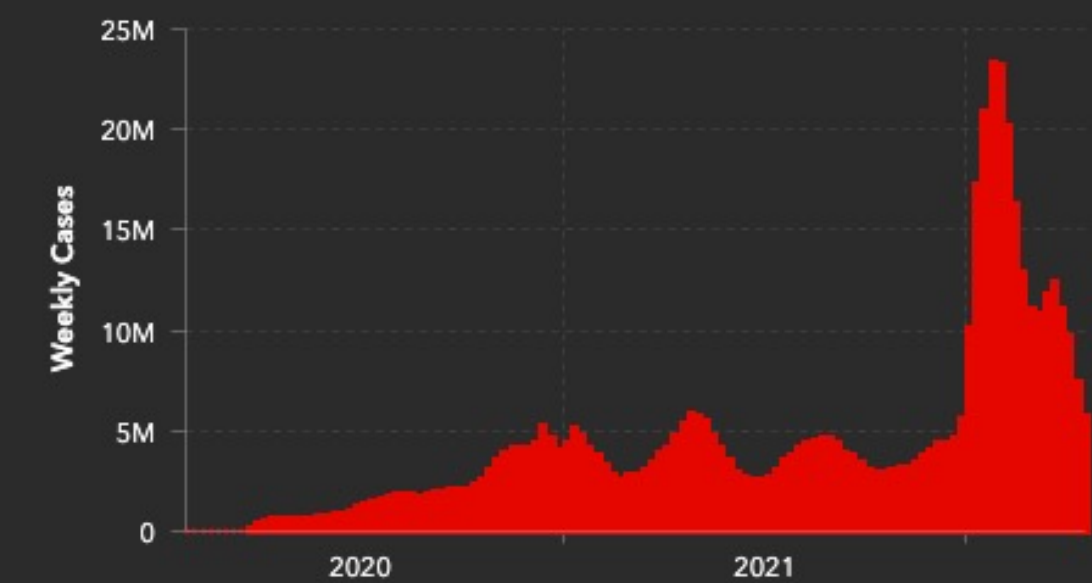
28-Day Vaccine Doses Administered

394,560,710



Esri, FAO, NOAA

Powered by Esri



Admin0

Admin1

Admin2

28-Day

Totals

Incidence

Case-Fatality Ratio

Global Vaccinations

US Vaccinations

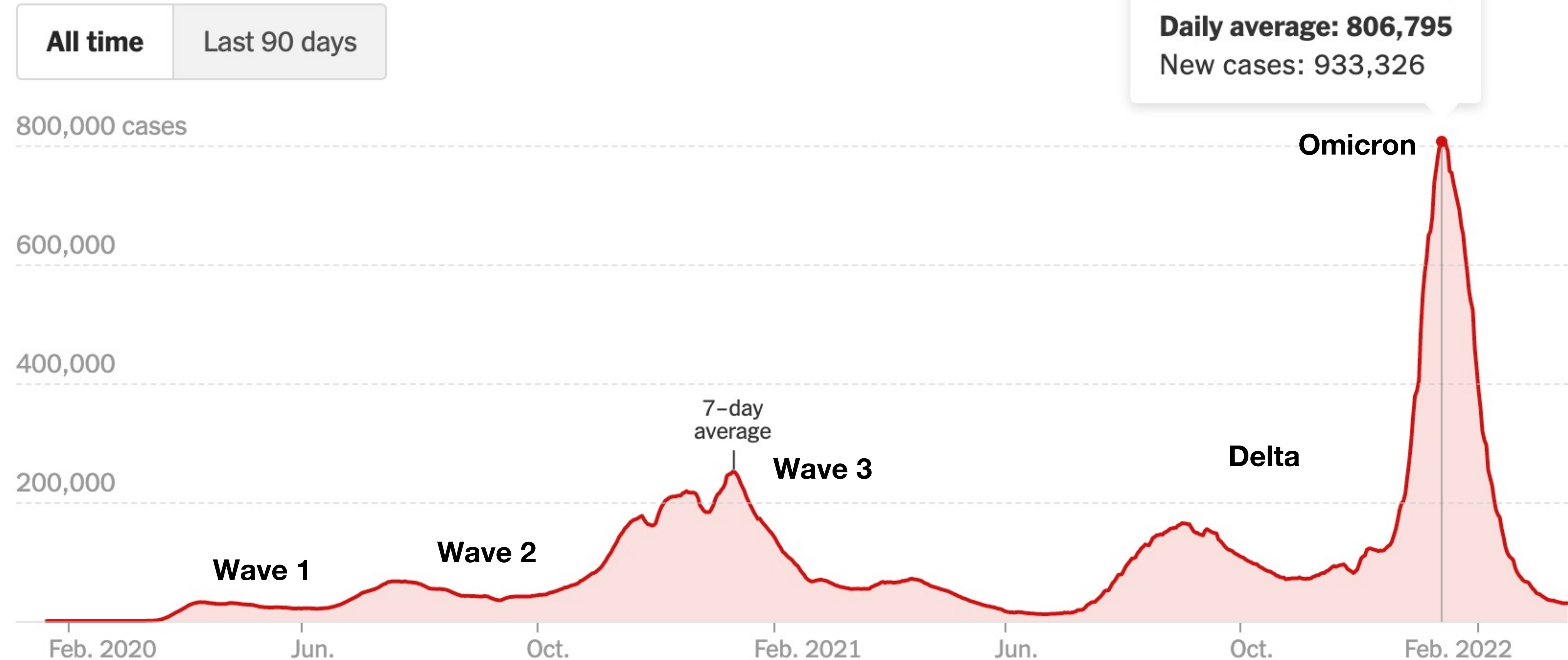
Terms of Use

Weekly

28-Day



# New reported cases



Source: NYTimes, <https://www.nytimes.com/interactive/2021/us/covid-cases.html>

Total in US ~ 80 million (Apr 21)

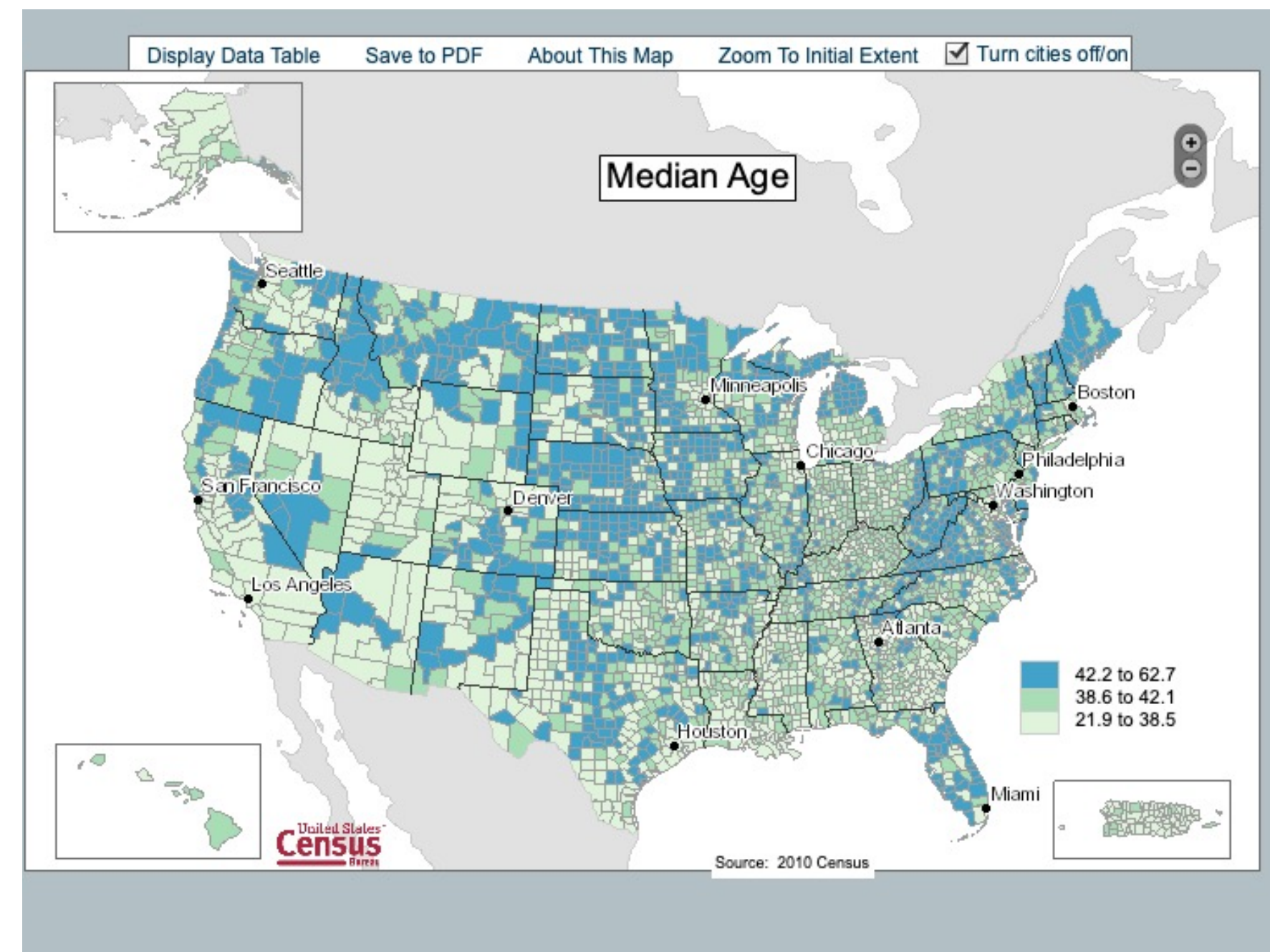
But how many unreported?



# NIH Serosurvey (Kaitlyn Sadtler, NIBIB)



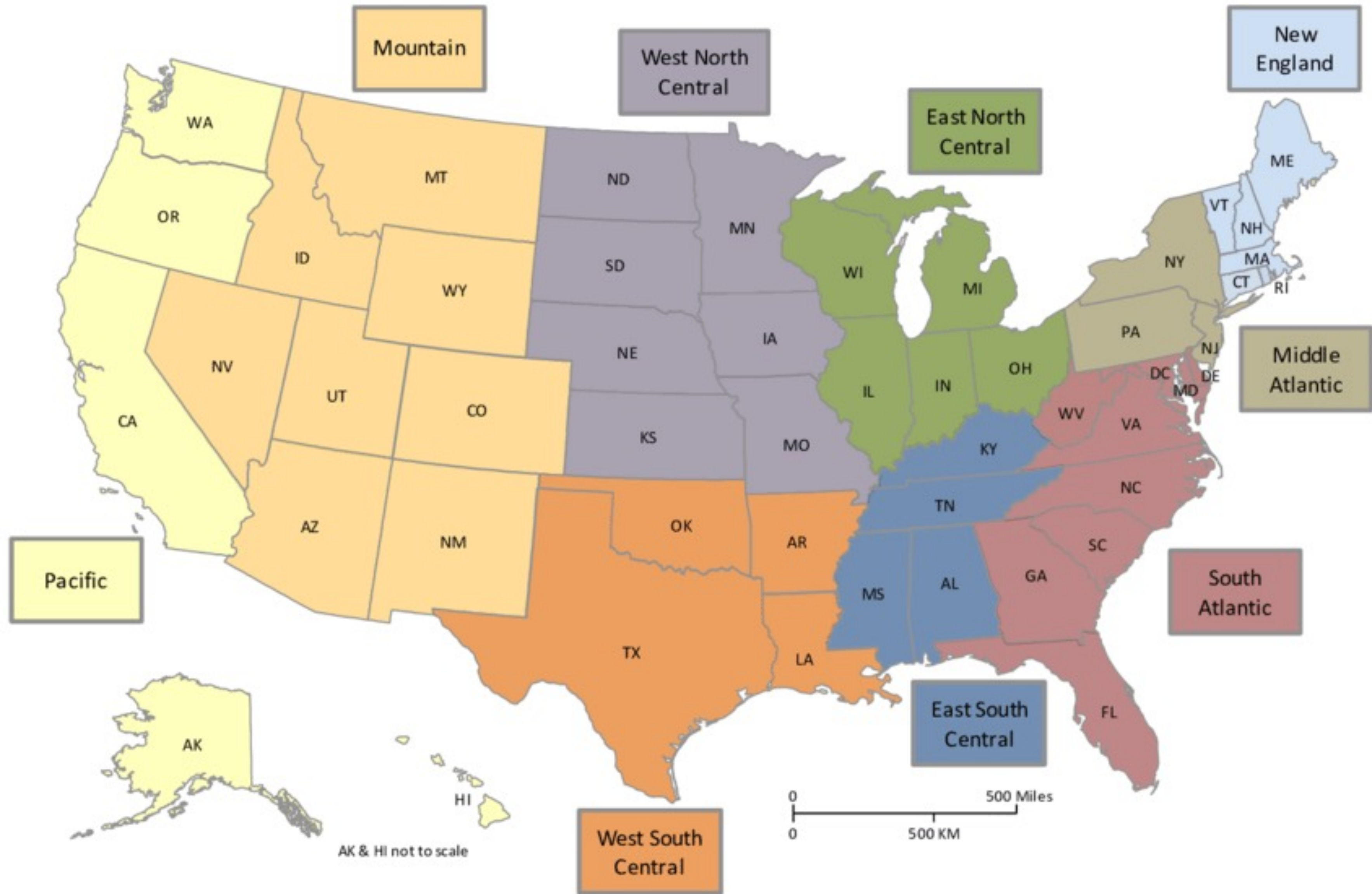
Neoteryx Mitra Volumetric Blood Sampling Kit  
(at-home dried blood)



Quota-Based Sampling from large volunteer pool  
(Goal:  $n = 10,000$ )

Participating IC's & Sites: NIBIB, NCATS, NIAID, FNLCR, UPitt, UAB





Can't assay everyone all the time

Extrapolate with a model

# Modeling a pandemic is hard

Biology is complicated

Data - incomplete, misreporting, changing standards

Network/spatial effects - not homogeneous

Population dependence, e.g. age, comorbidities, genetics, history

Physiology - duration and severity

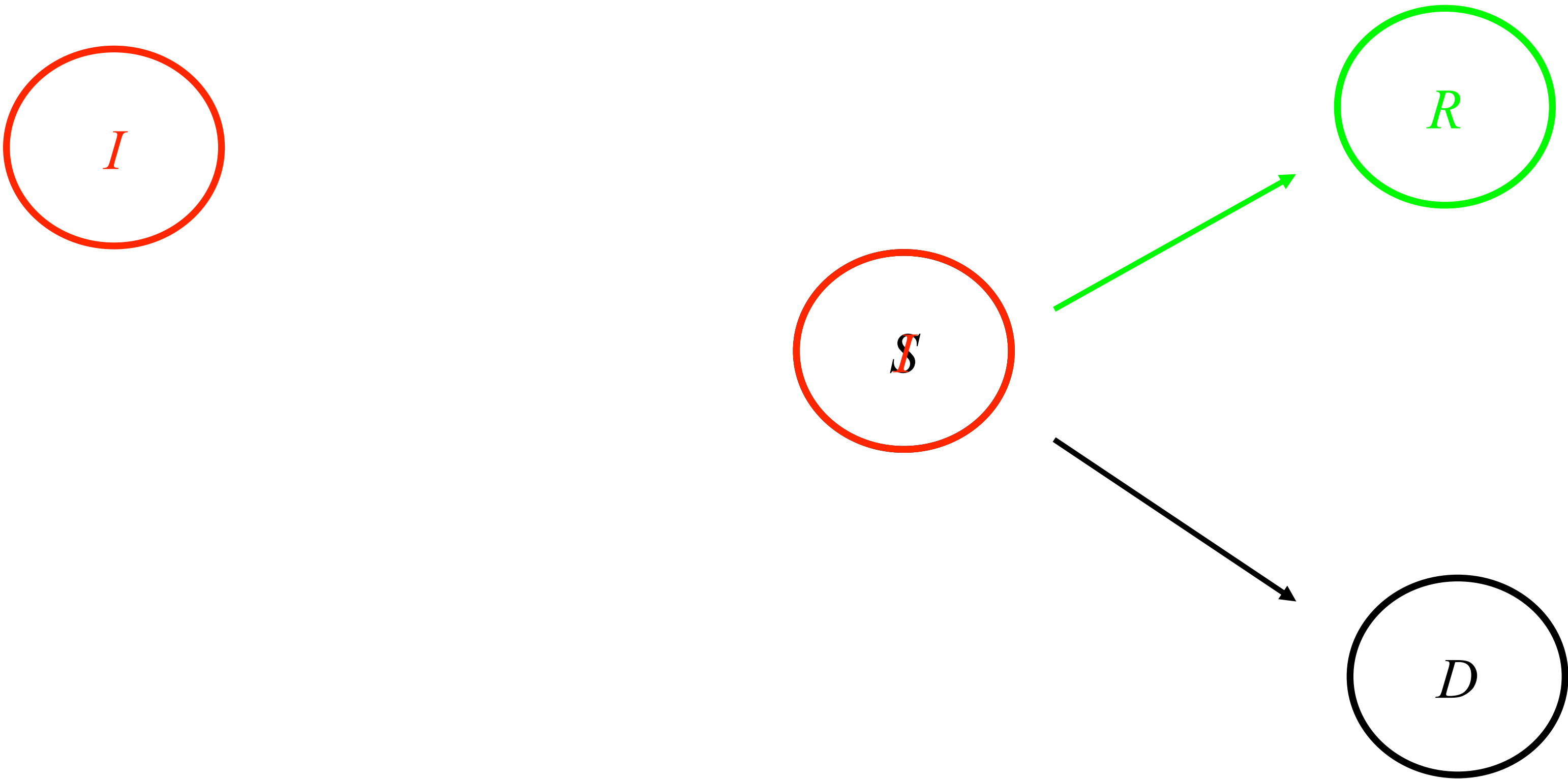
Behavior and policy

What should a model include?



Minimal model

SIR(D)



Network dependent birth-death stochastic process

# Mass Action SIR

$$\frac{dI}{dt} = \frac{\beta}{N}SI - \sigma I$$

$$\frac{dS}{dt} = -\frac{\beta}{N}SI$$

$$\frac{dR}{dt} = \sigma_R I$$

$$\frac{dD}{dt} = \sigma_D I$$

$$\sigma = \sigma_R + \sigma_D$$

# Mass Action SIR

$$\frac{dI}{dt} = \beta SI - \sigma I$$

$$\frac{dS}{dt} = -\beta SI$$

Scale by N

I peaks at  
(I nullcline)

$$\frac{dI}{dt} = 0 = \beta SI - \sigma I$$

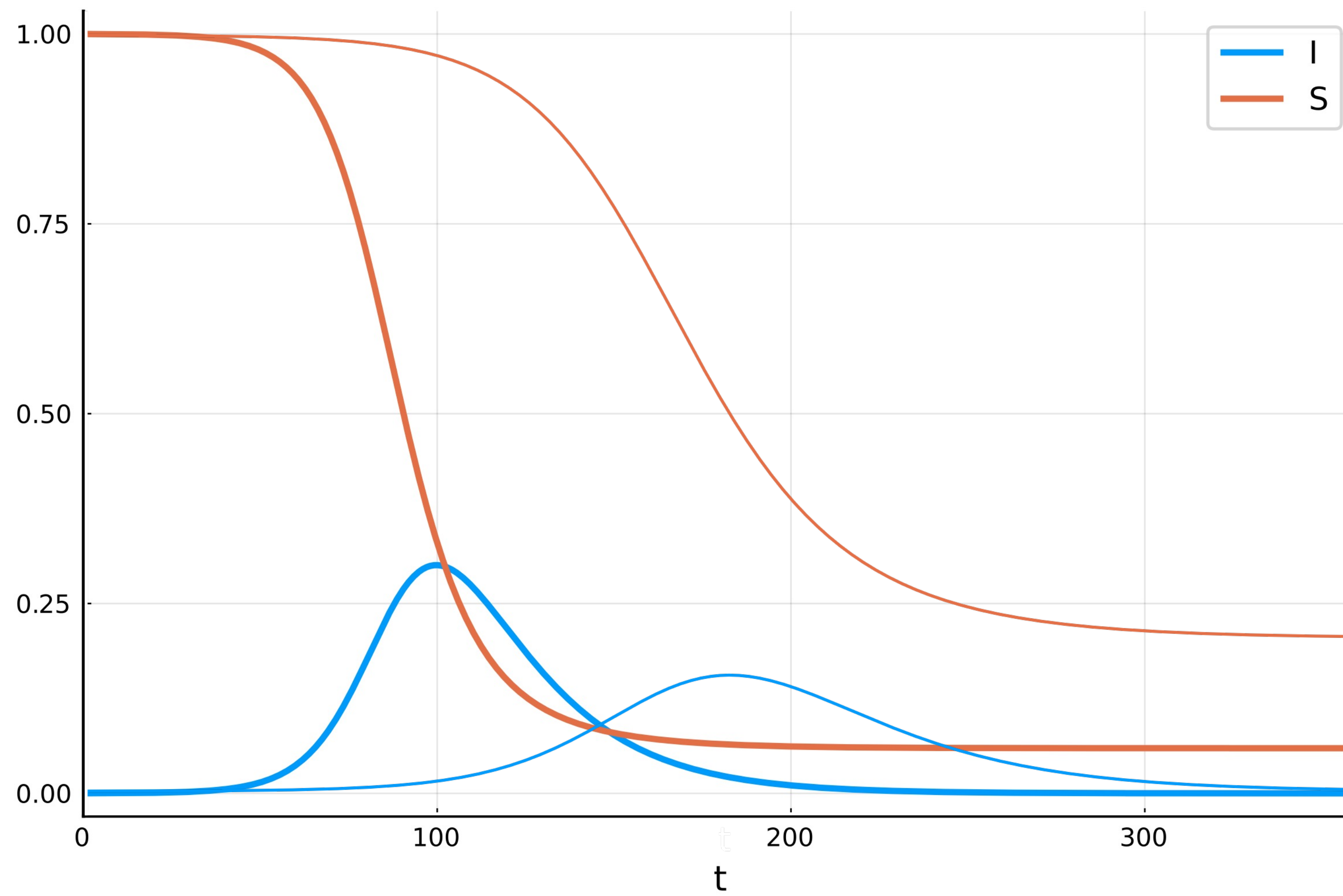
$$S^* = \frac{\sigma}{\beta} = \frac{1}{R_0}$$

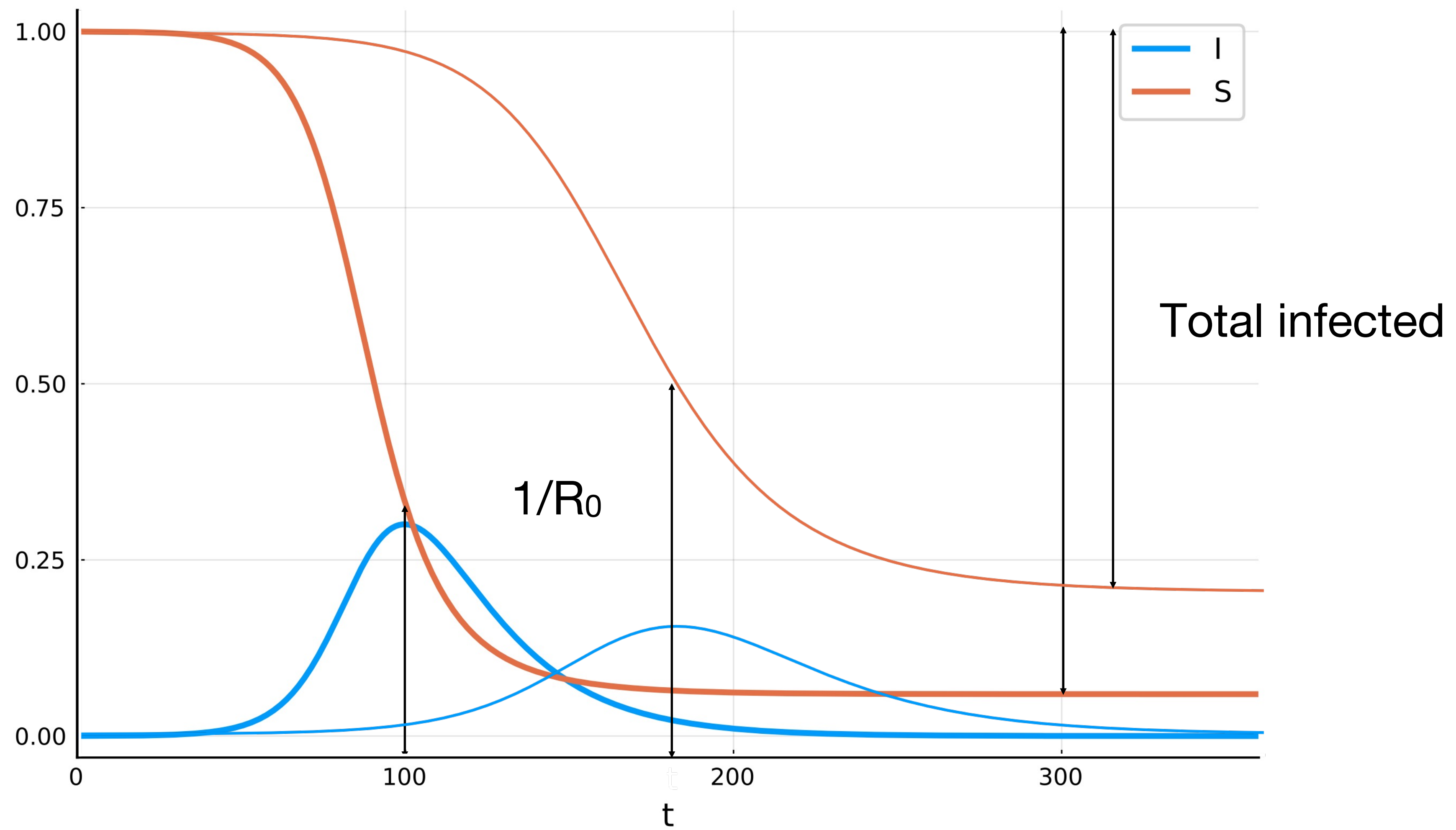
Herd immunity  
threshold

$R_0$  = initial reproduction number

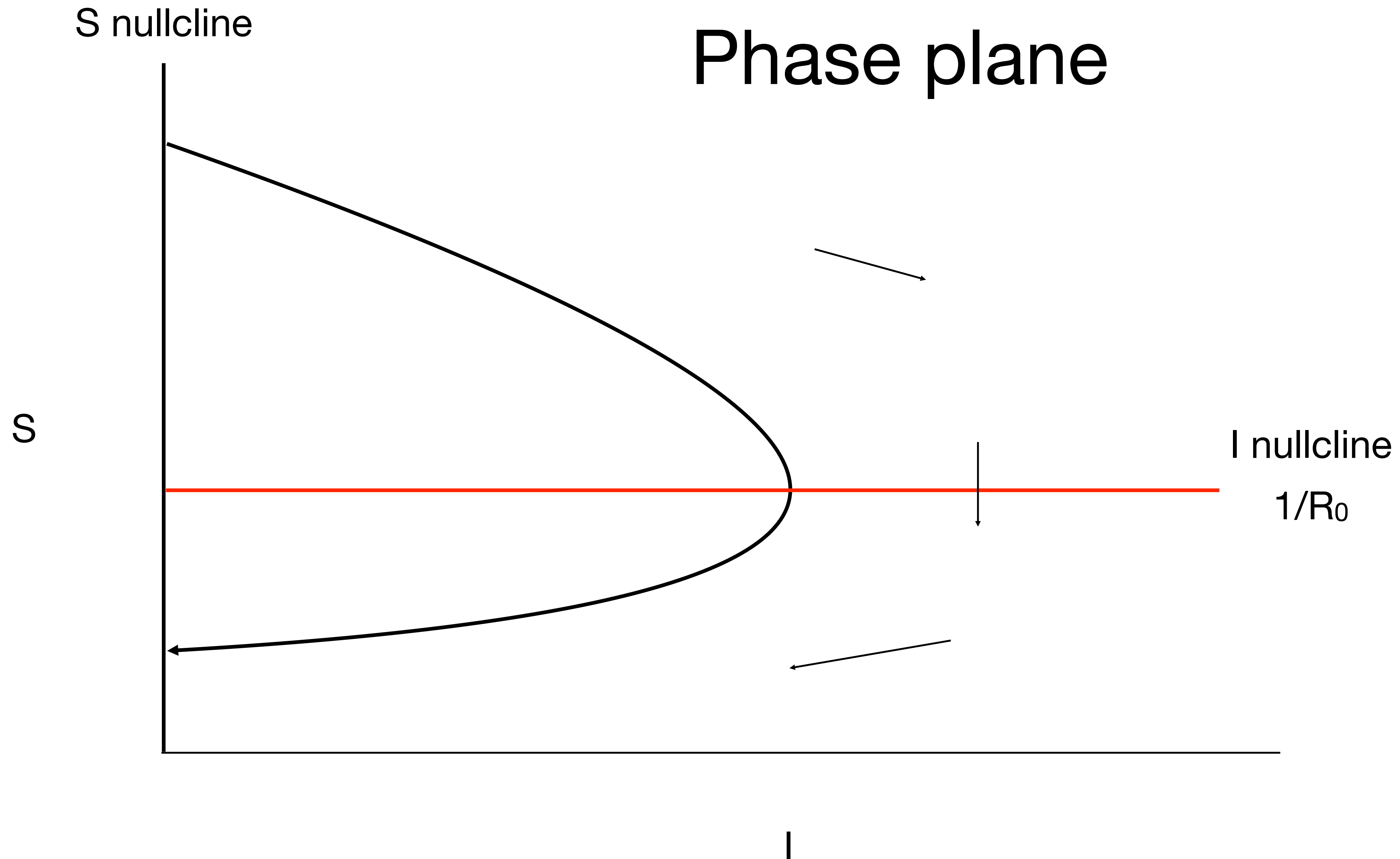


$$\beta = 0.15, \sigma = 0.05, R_0 = 23$$





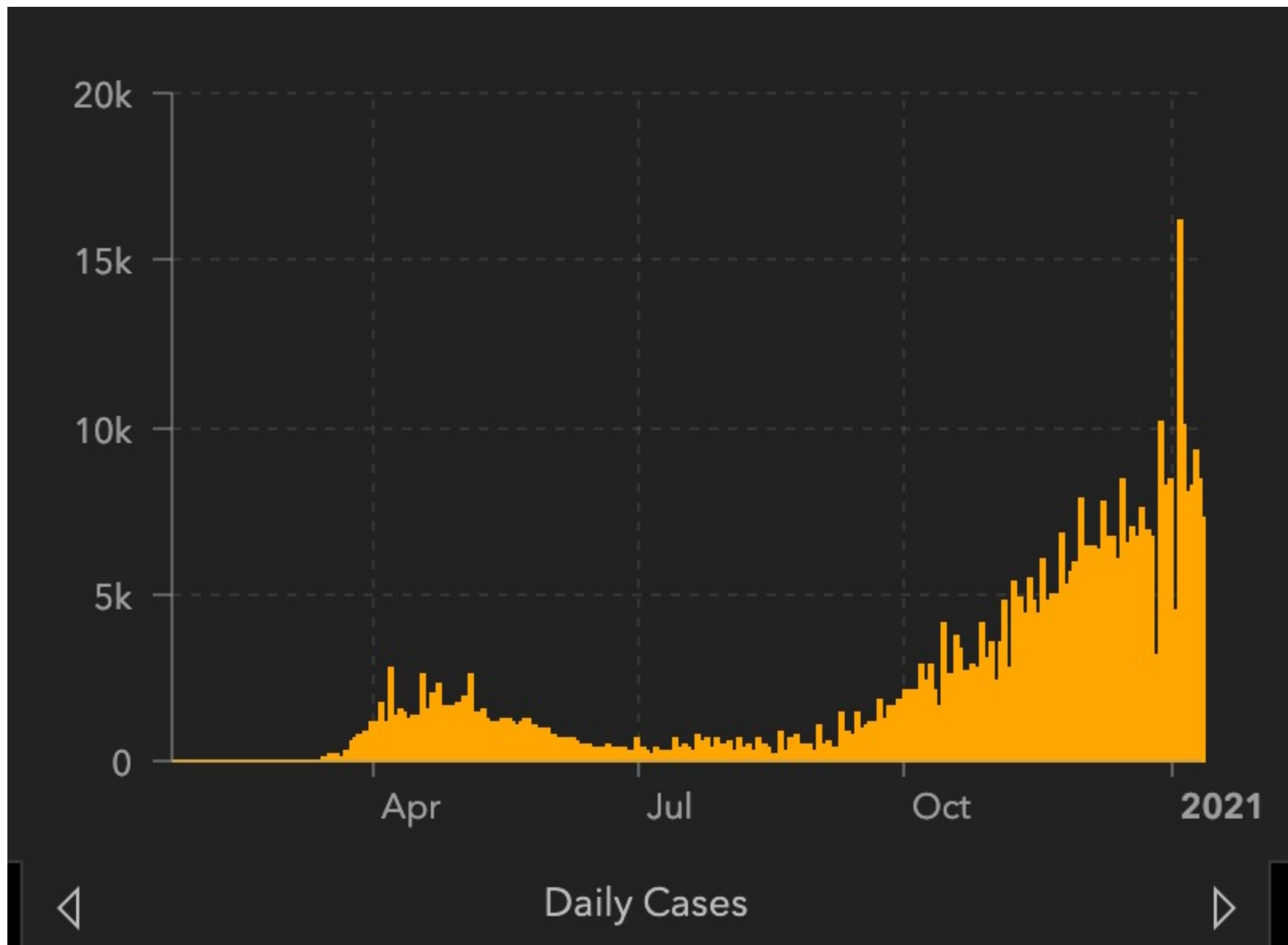
# Phase plane



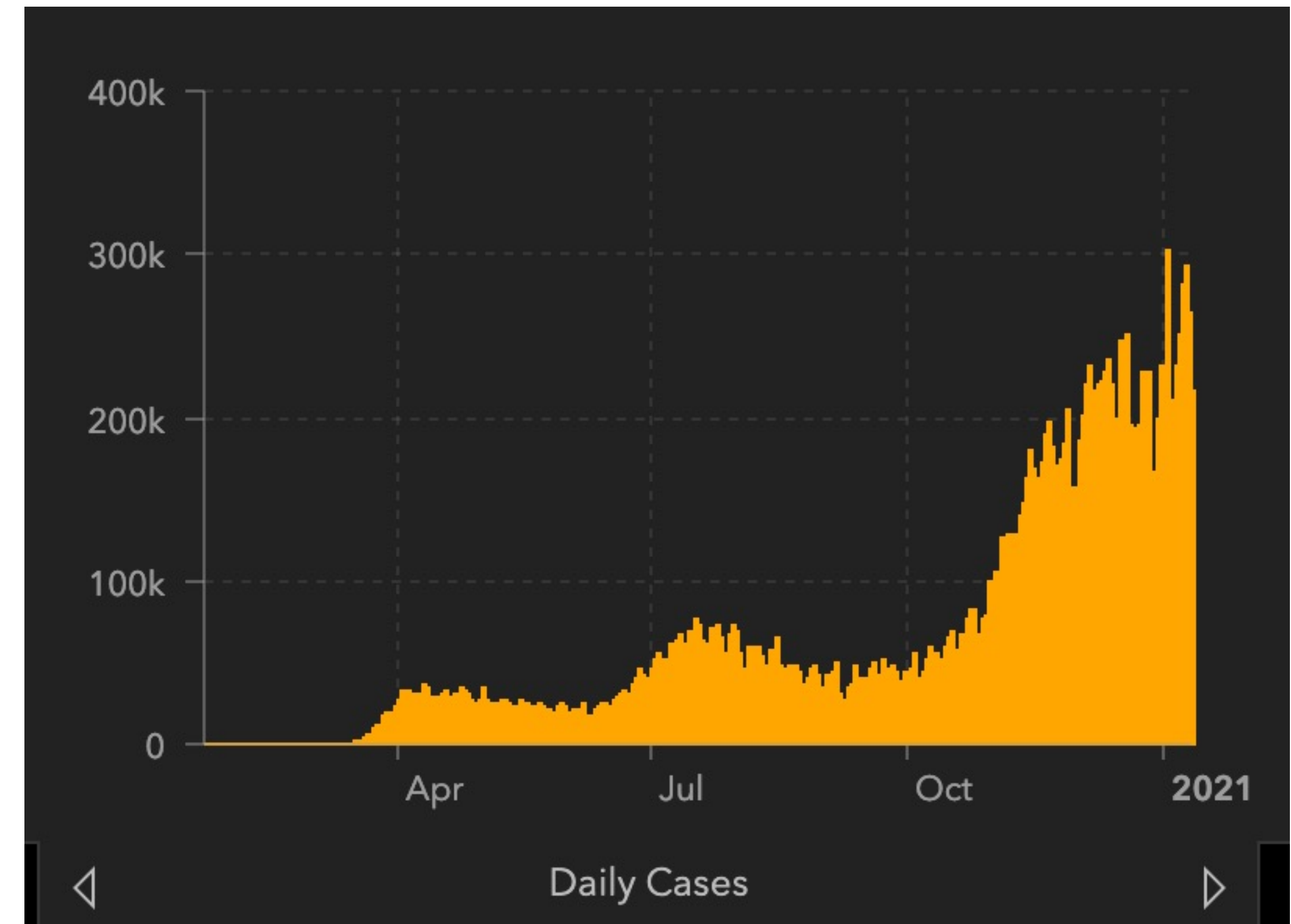


# Recurrence and plateaus

Canada

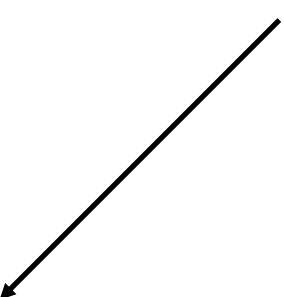


USA



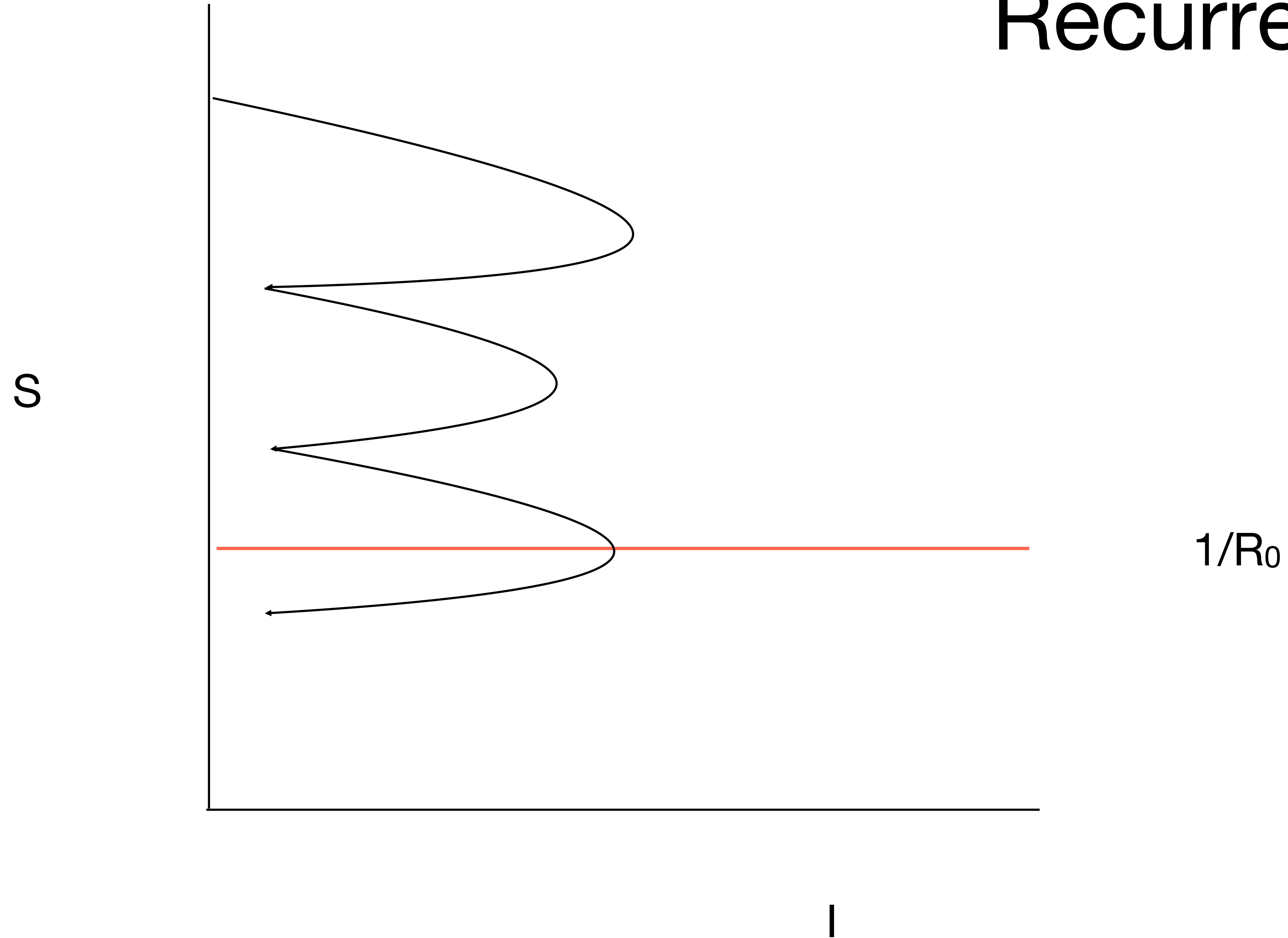
# Time dependence

Policy, behavior, viral evolution, ...


$$\frac{dI}{dt} = \beta(t)SI - \sigma I \quad \frac{dS}{dt} = -\beta(t)SI$$

Move the nullcline

# Recurrence





# Plateaus

$$\frac{dI}{dt} = \beta(t)SI - \sigma I$$

$$\frac{dS}{dt} = -\beta(t)SI$$

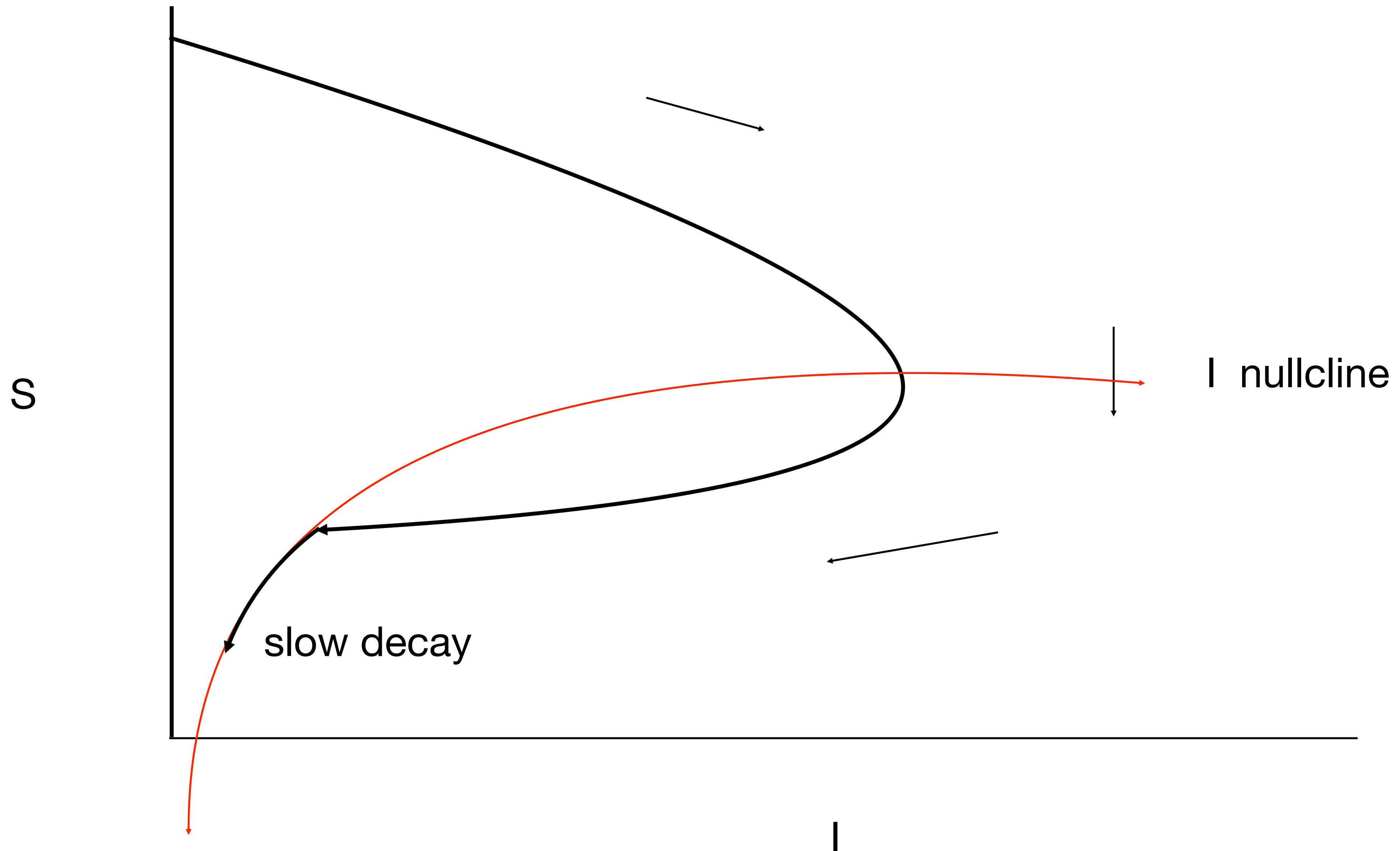
$$\frac{dI}{dt} = \beta(t)SI - \sigma I = 0$$

Unstable equilibrium

# Plateaus

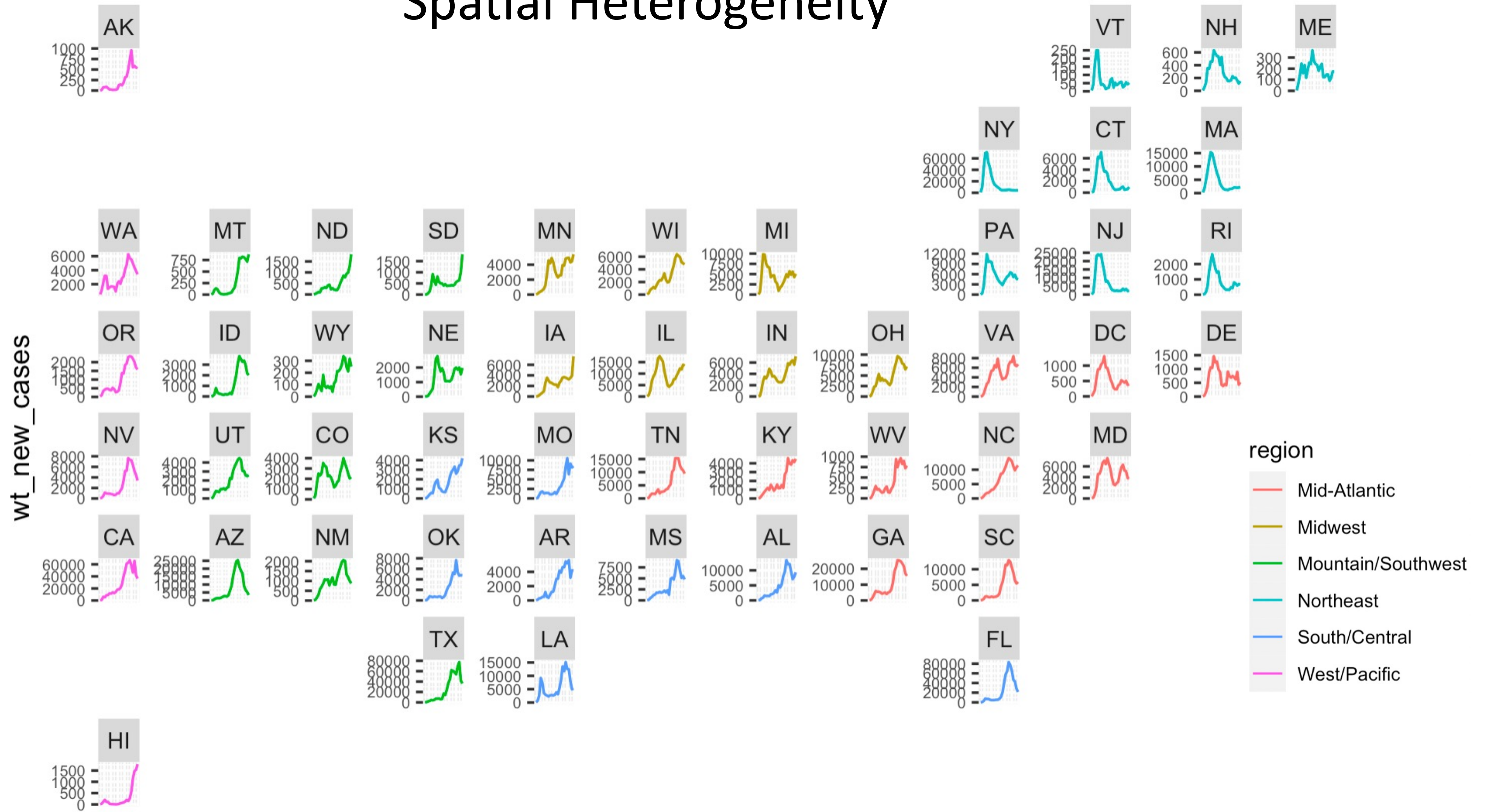
$$\frac{dI}{dt} = \beta(t)SI - \sigma I + \alpha \quad \text{migration} \quad \frac{dS}{dt} = -\beta(t)SI$$

$$S = \frac{\sigma I - \alpha}{\beta I} \quad \text{Quasi-stable equilibrium}$$





# Spatial Heterogeneity



Weekly totals spanning from March 8, 2020 to September 4, 2020.

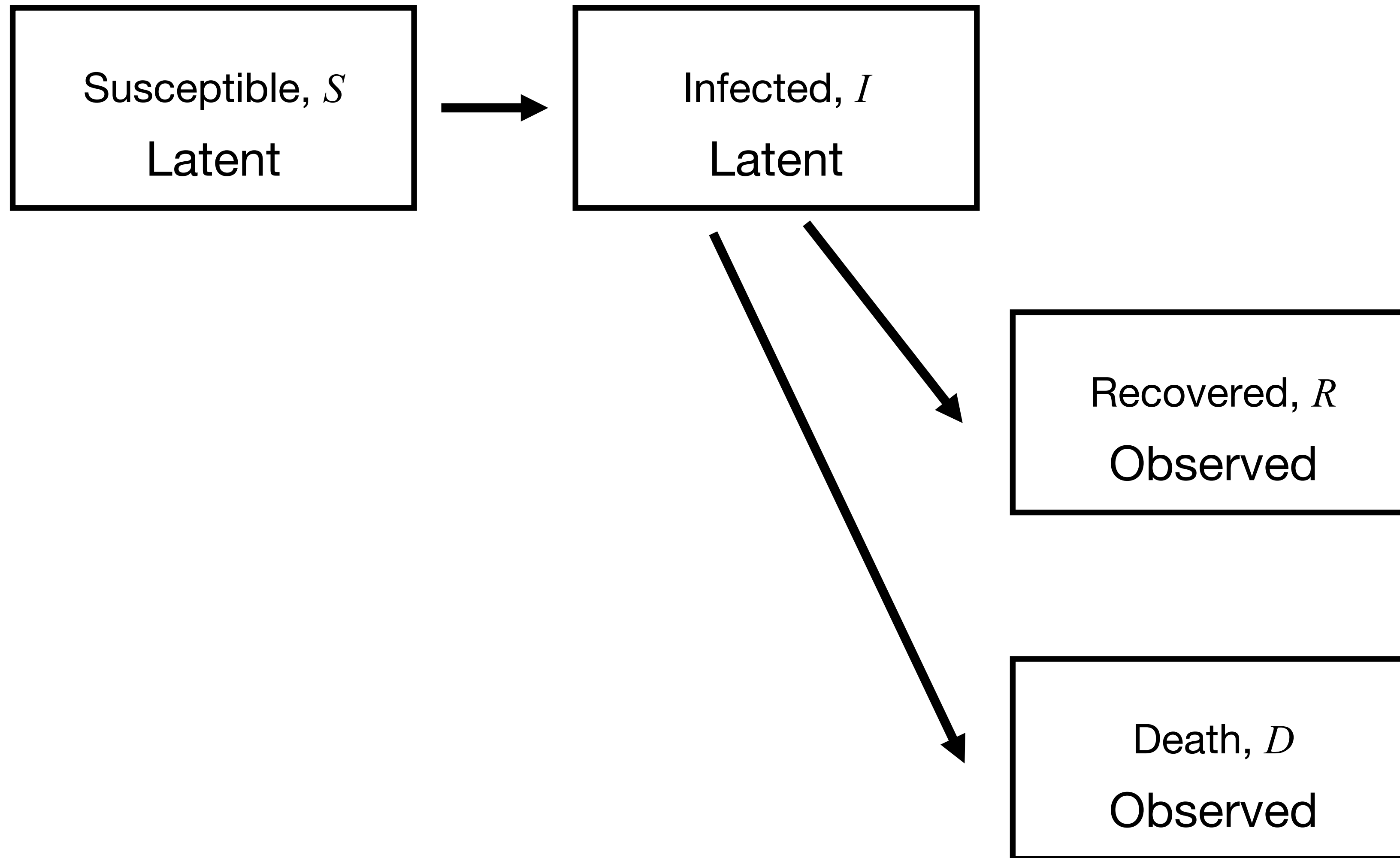
Model needs to include at least:

Time dependent infection rate, migration, spatial inhomogeneity

Reported cases are a fraction of total infected

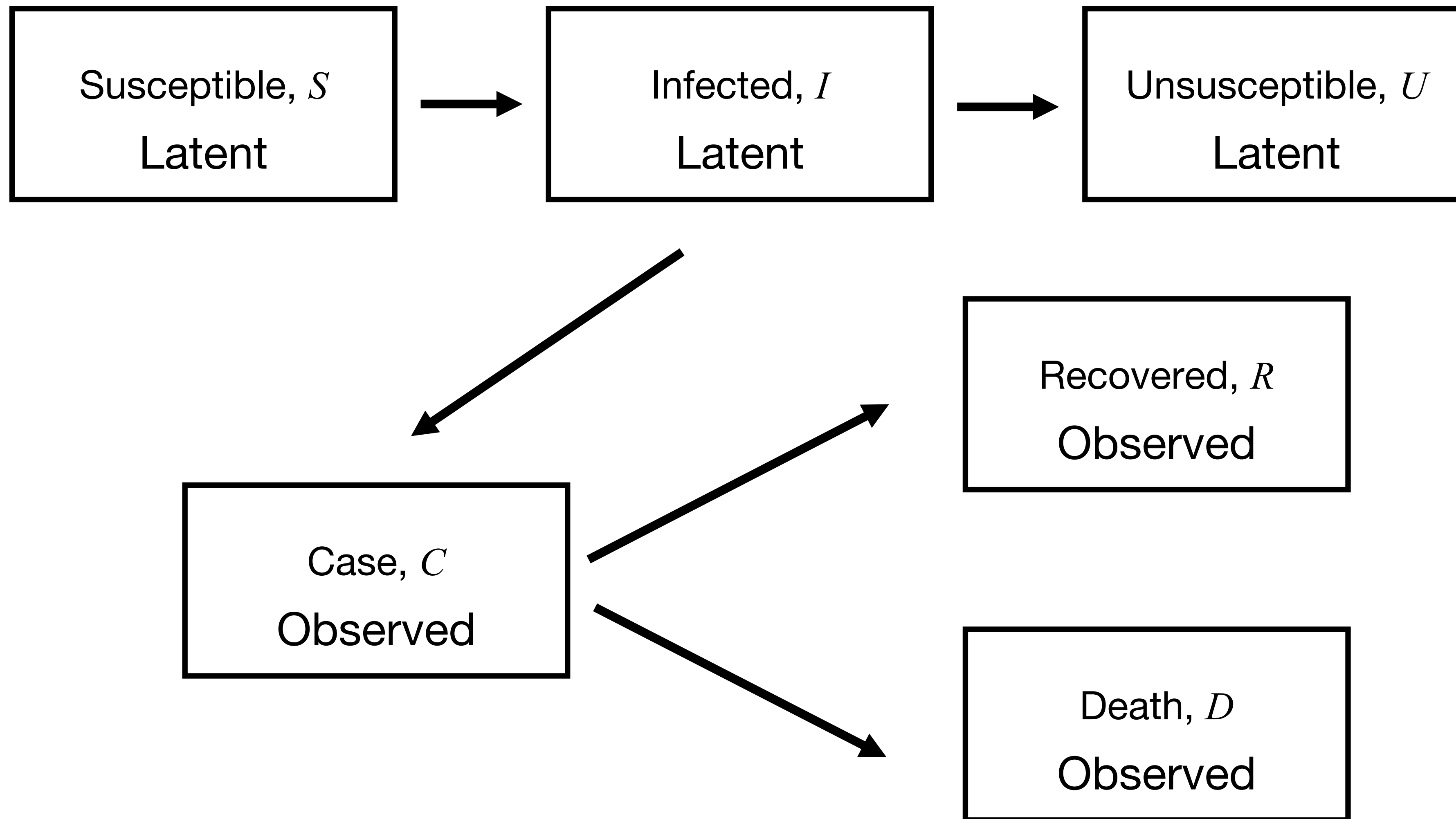
Case ascertainment ratio = total cases/total infections

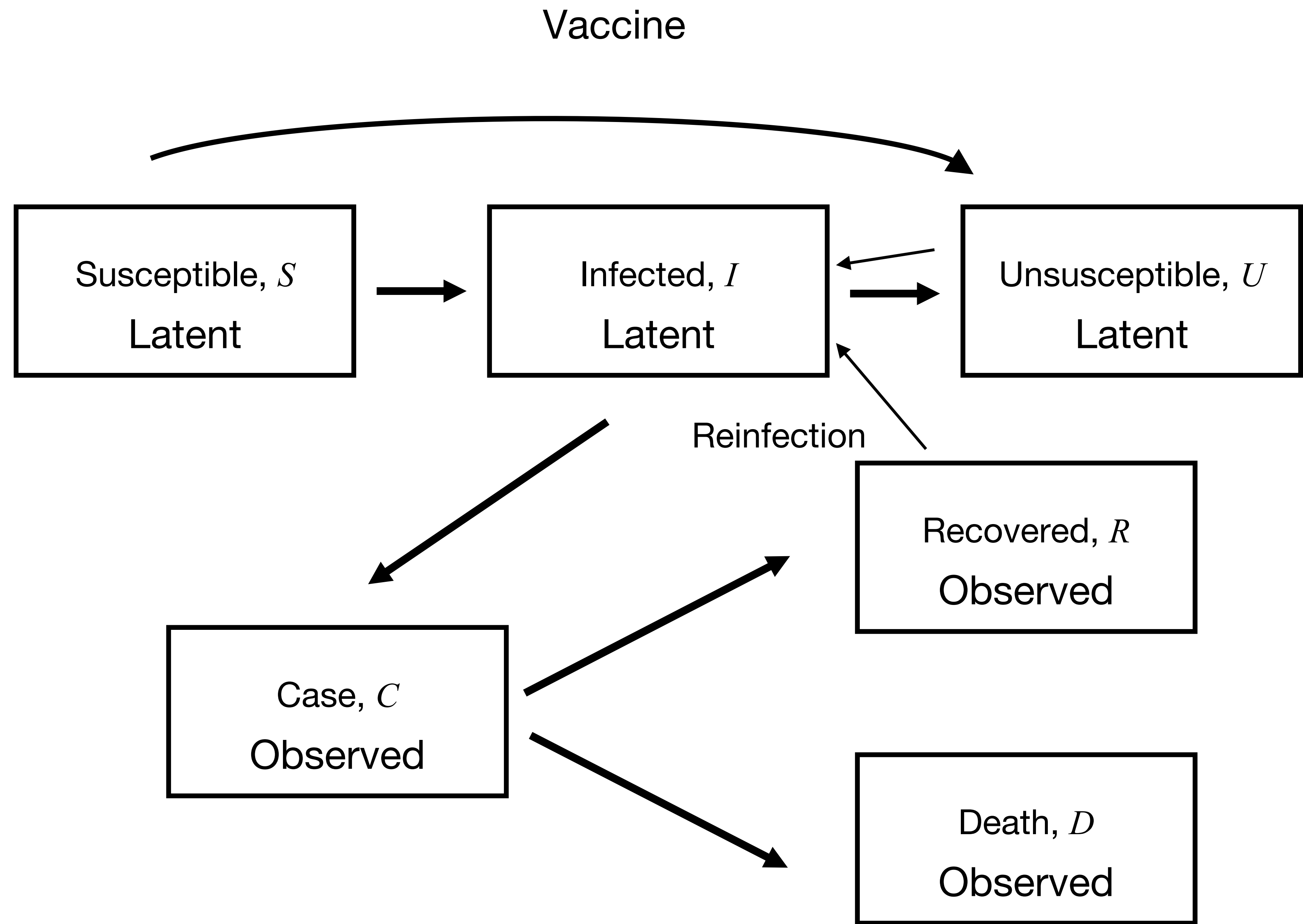
# SIR model





# Latent variable SICR model





# Measures

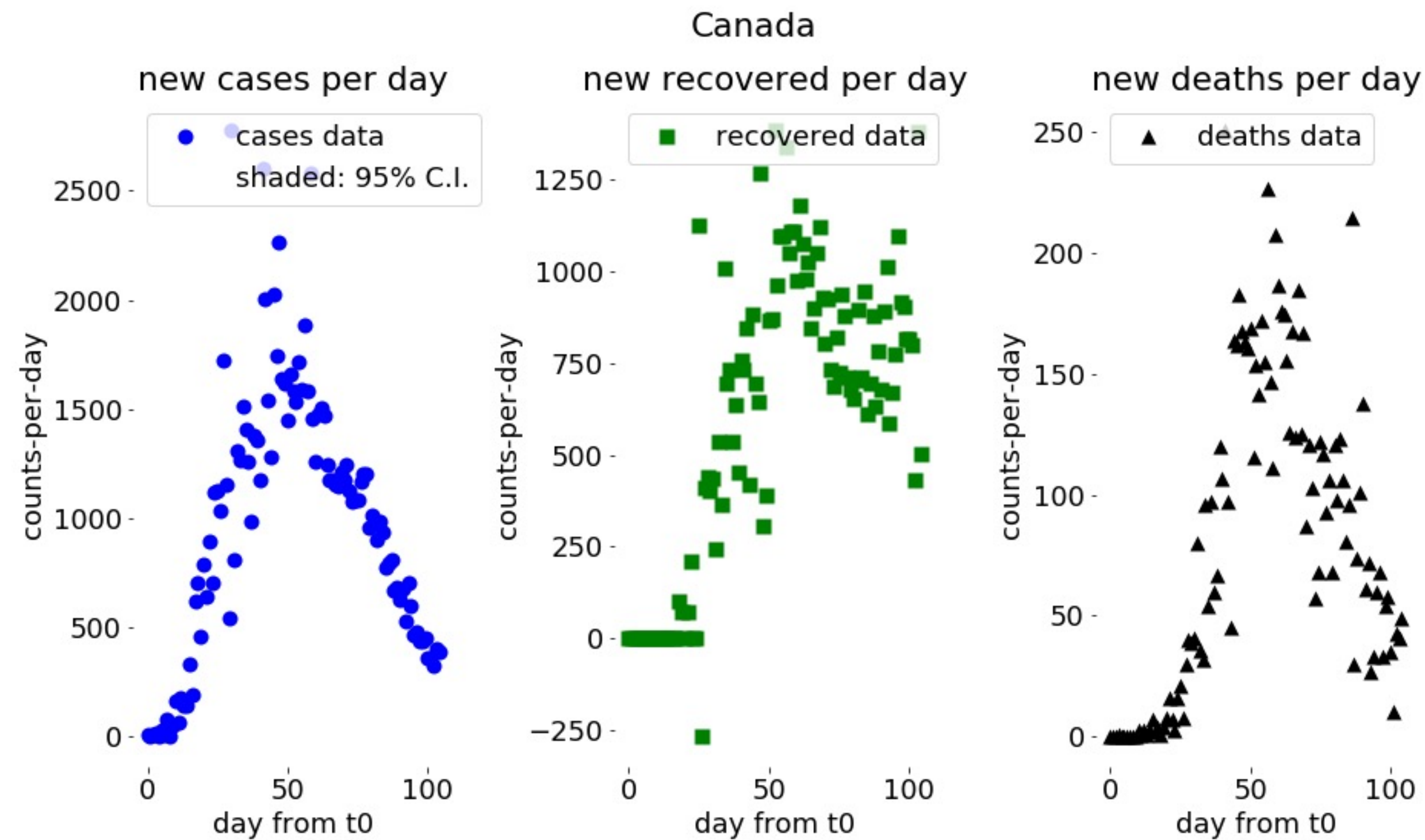
$CAR_t = \text{total cases} / \text{total infected}$

$IR_t = \text{total infected} / \text{total population}$

$R_t = \text{time dependent reproduction number}$

# Bayesian Inference

Fitting models in the presence of data and parameter uncertainty



# Bayesian Inference

Statistics by applying the rules of probability

$$P(A, B) = P(A|B)P(B) = P(B|A)P(A)$$

$$P(A) = \sum_B P(A|B)P(B)$$

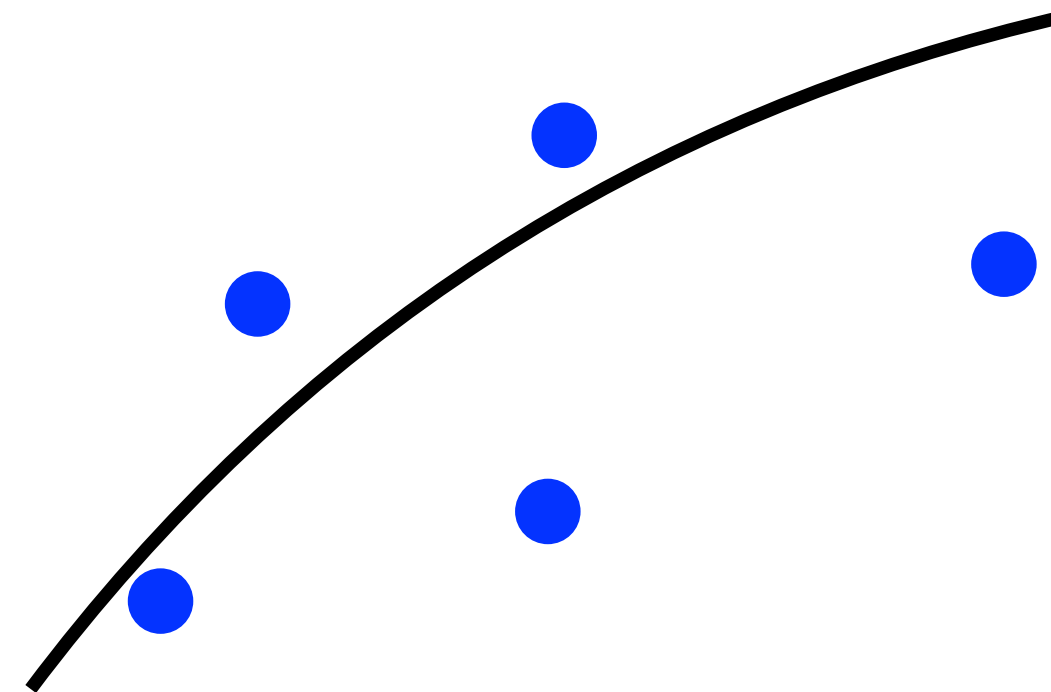


# Bayesian Inference

Fitting models in the presence of data and parameter uncertainty

Data has noise

$$D \sim P(D|y(\theta))$$



Model depends on parameters

$$y(\theta)$$

e.g. Likelihood:  $P(D|\theta) \propto e^{-(D-\bar{y})^2/2\sigma^2} \equiv P(D|y(\theta))$

# Bayesian Inference

Fitting models in the presence of data and parameter uncertainty

Prior

Parameter posterior

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} = \frac{P(D|\theta)P(\theta)}{\int P(D|\theta)P(\theta)d\theta}$$

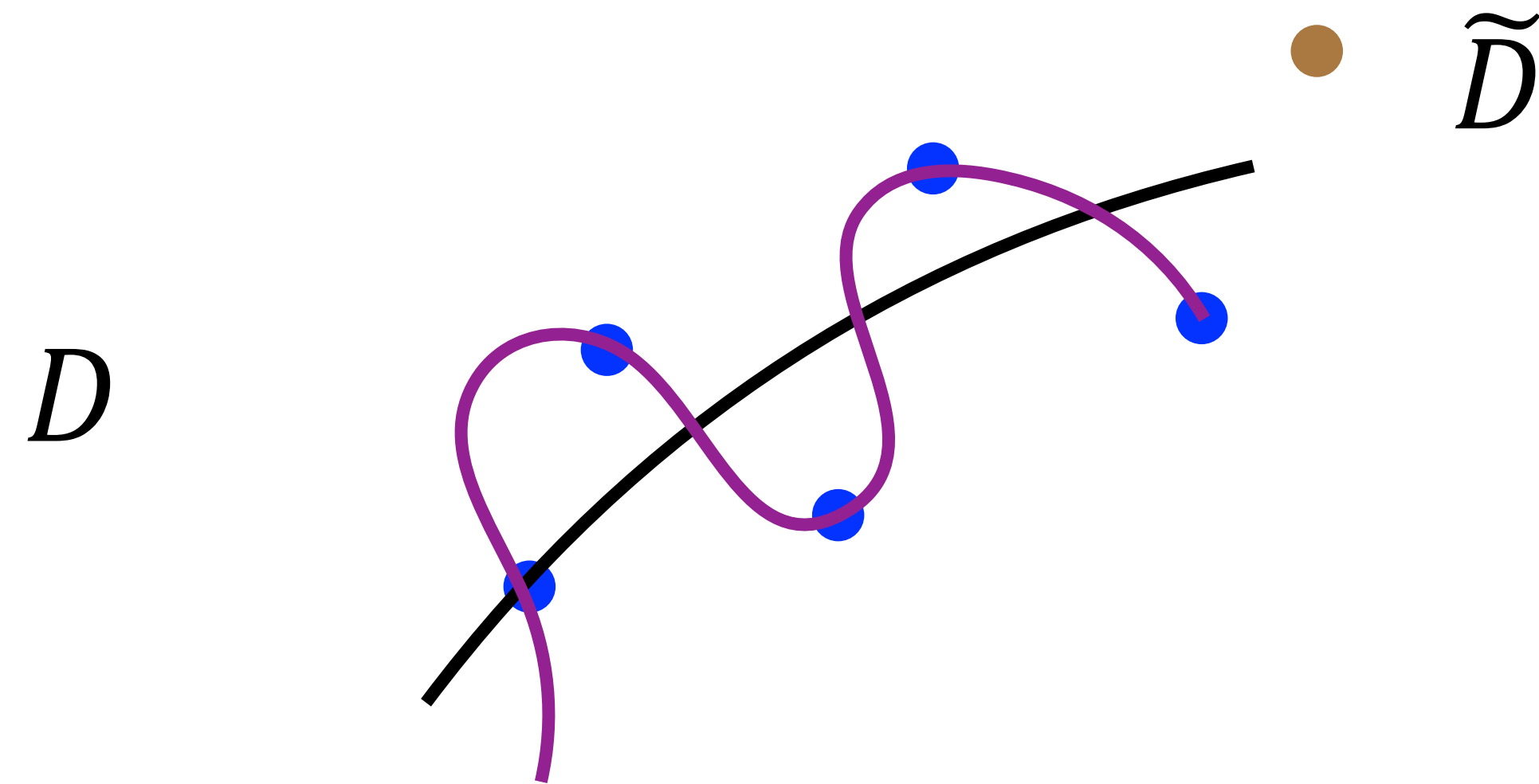
Projections

$$P(\tilde{D}|D) = \int P(\tilde{D}|\theta)P(\theta|D)d\theta$$

# Bayesian model comparison and model averaging

Try lots of models,  $M$

Balance fit with complexity



$$P(M|D) = \frac{P(D|M)P(M)}{P(D)}$$

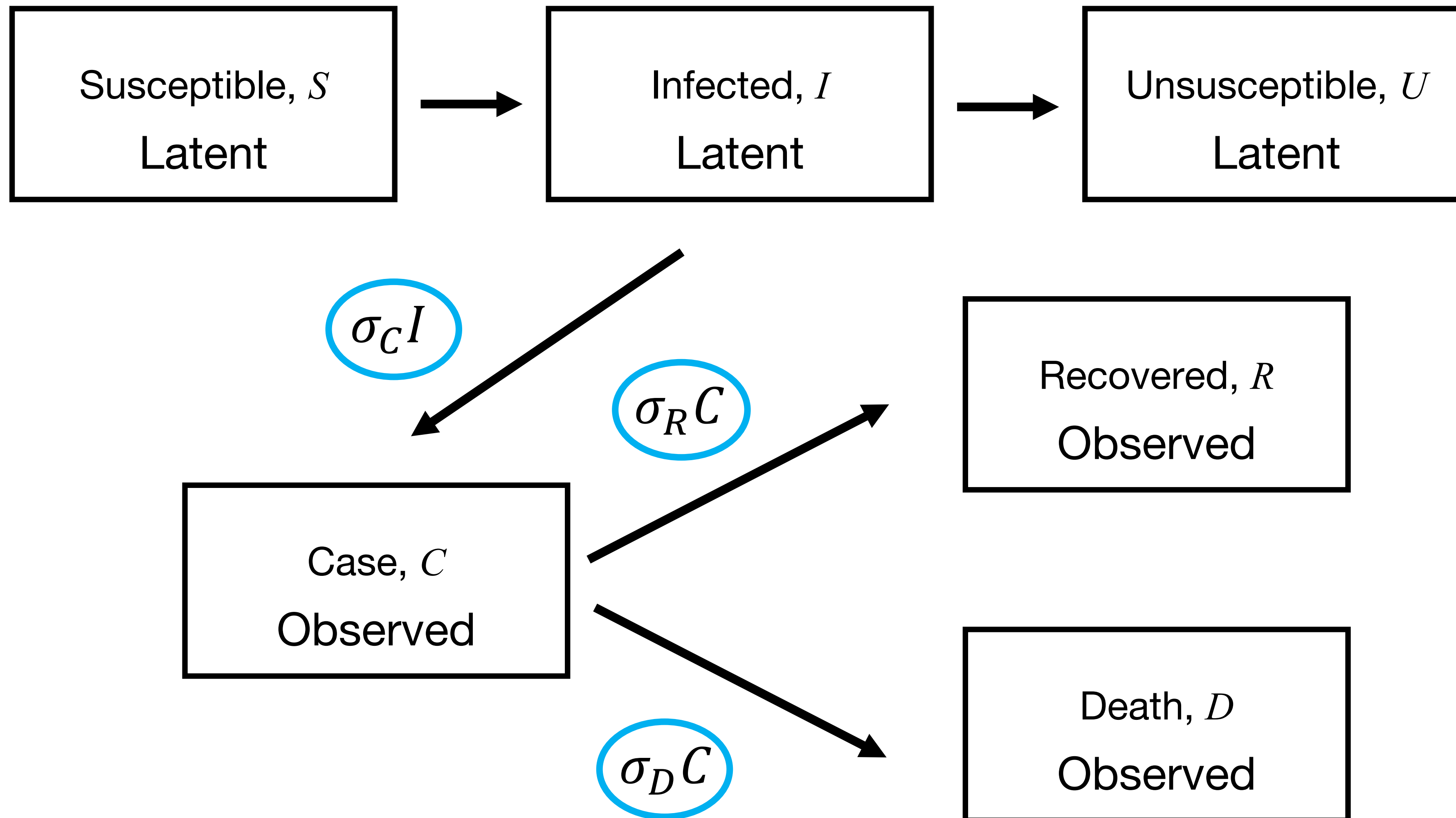
Maximize model likelihood  $P(D|M)$

Bayes factor

Maximize prediction  $P(\tilde{D}|M)$

cross-validation, Information measures,  
e.g. AIC, WAIC,...

# Pick the right data to fit



# Noise model

If events arrive randomly then counts are Poisson distributed

Add extra variance due to heterogeneity and misreporting

New cases / day  $\sim \text{NegBinom}(\sigma_c I, \phi)$

New recoveries / day  $\sim \text{NegBinom}(\sigma_R C, \phi)$

Daily deaths / day  $\sim \text{NegBinom}(\sigma_D C, \phi)$

$$var = \lambda + \frac{\lambda^2}{\phi}$$

Over dispersion compared to Poisson



Include as prior

NIH seropositivity estimates for May 9 - June 25, 2020

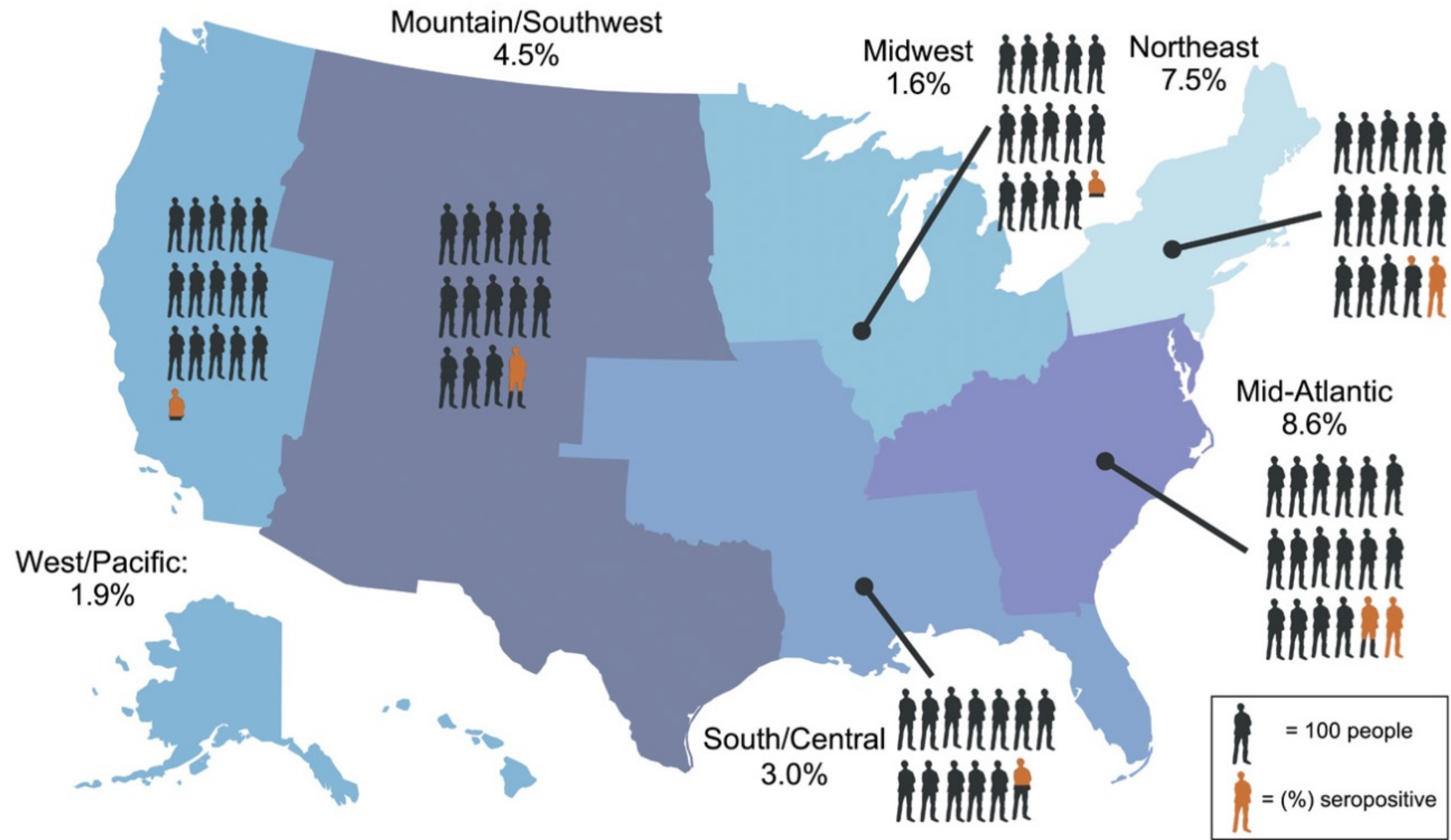


Figure from “Undiagnosed SARS-CoV-2 seropositivity during the first 6 months of the COVID-19 pandemic in the United States” by Heather Kalish et al, Science Translational Medicine <https://www.science.org/doi/10.1126/scitranslmed.abh3826>

# Hierarchical (mixed) model using Stan

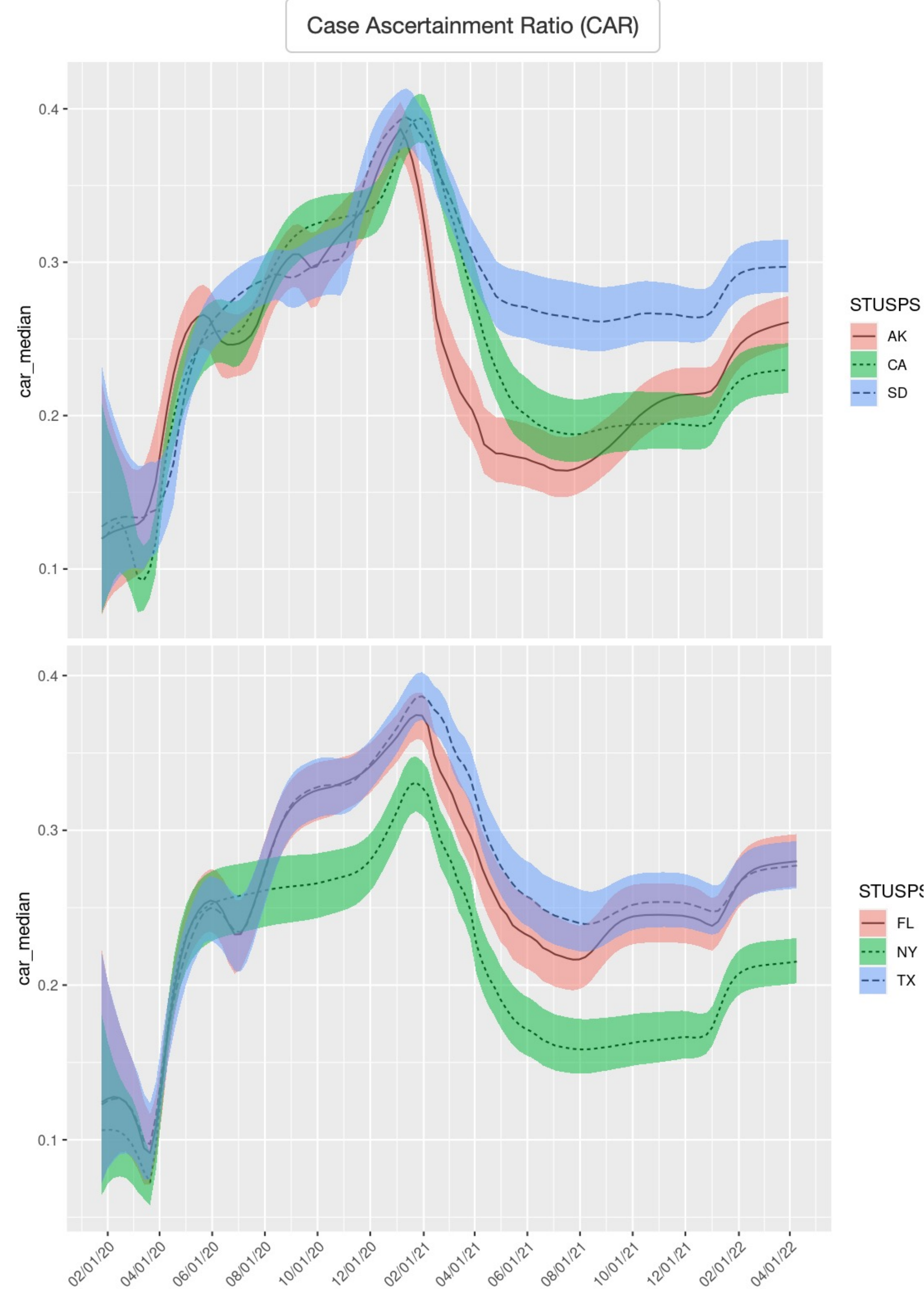
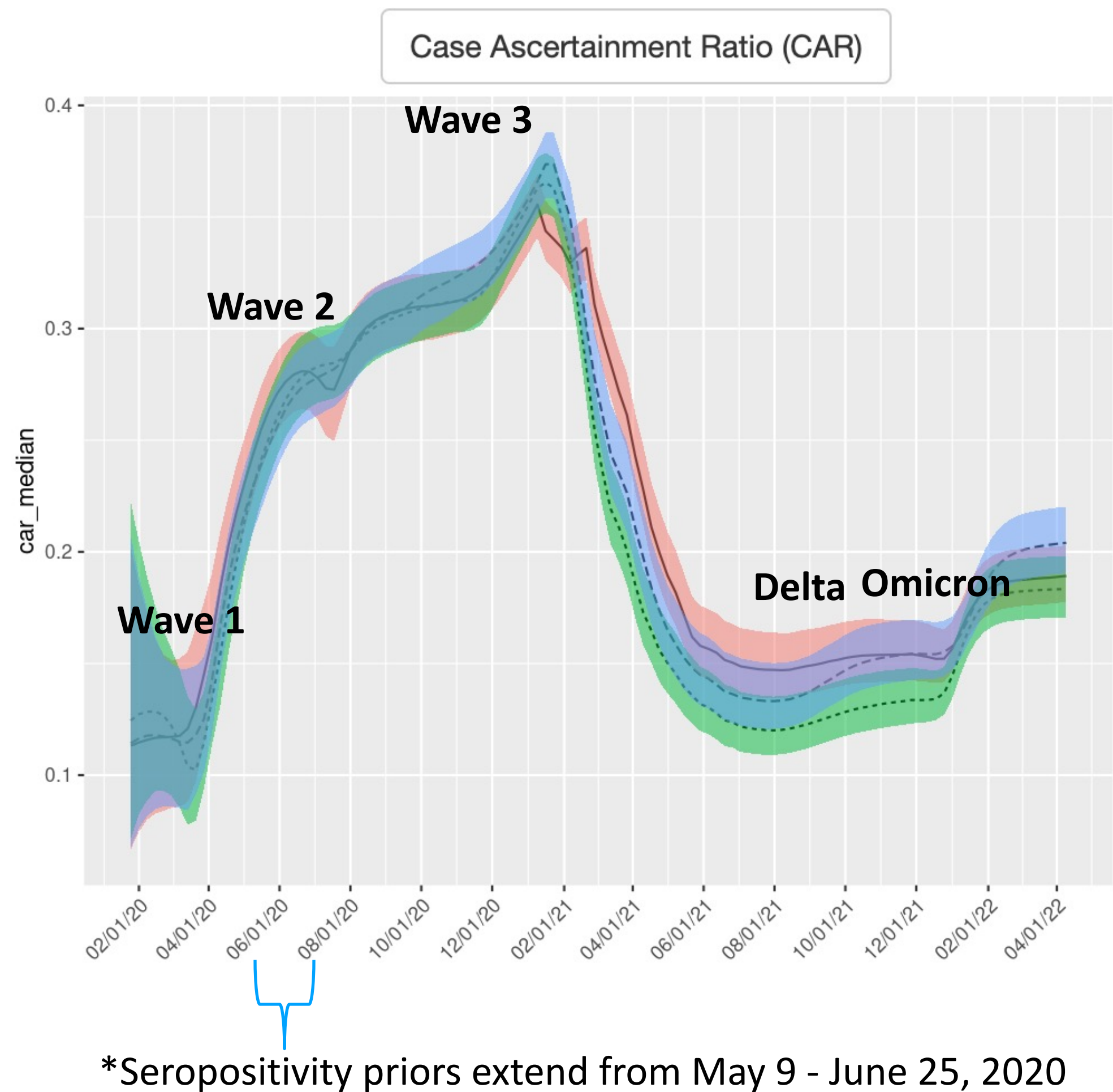
<https://mc-stan.org/>

Markov Chain Monte Carlo (NUTS)

<https://github.com/nih-niddk-mbs/covid-sicr/tree/Hierarchical>

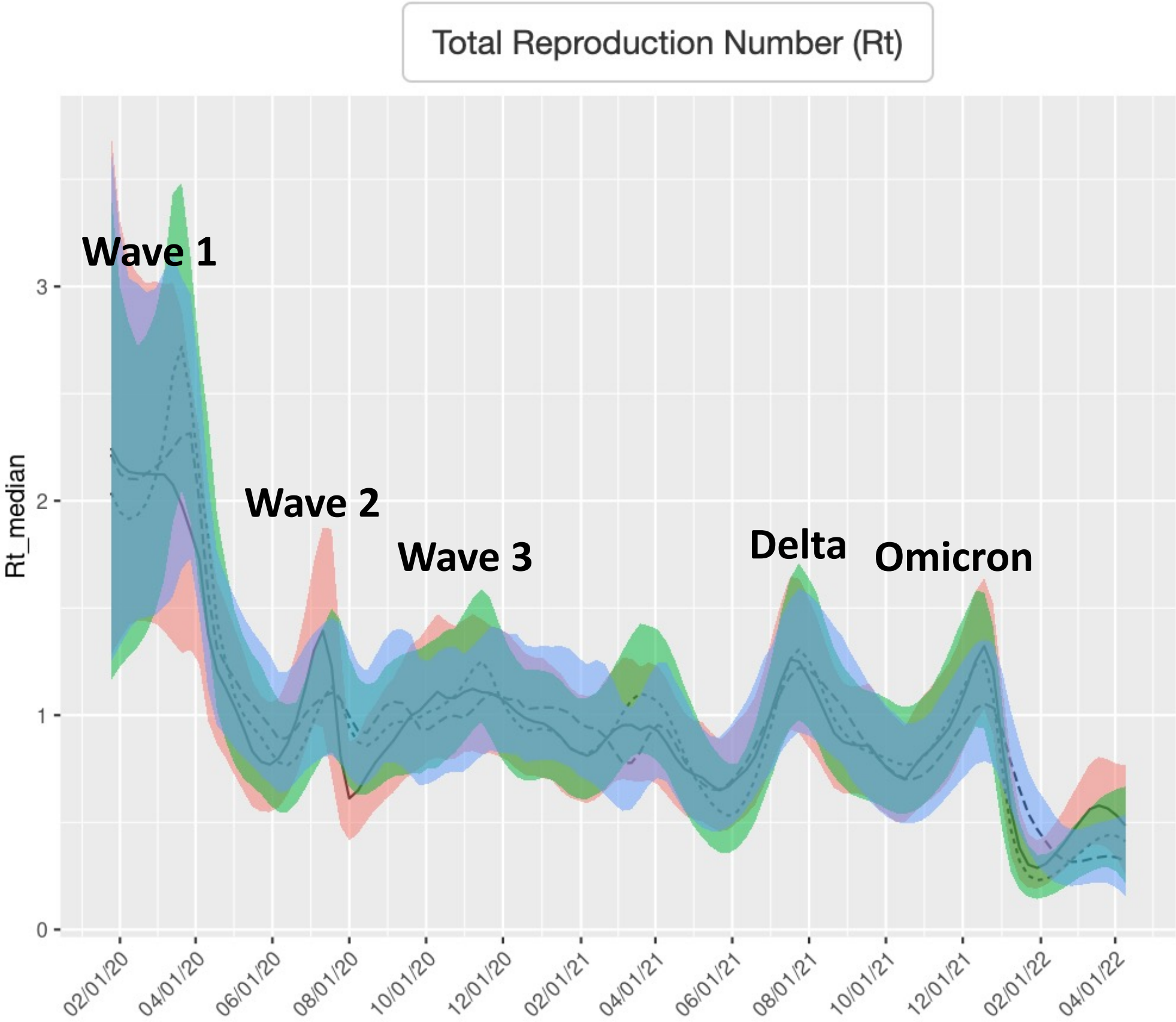


# Case Ascertainment Ratio (CAR<sub>t</sub>)



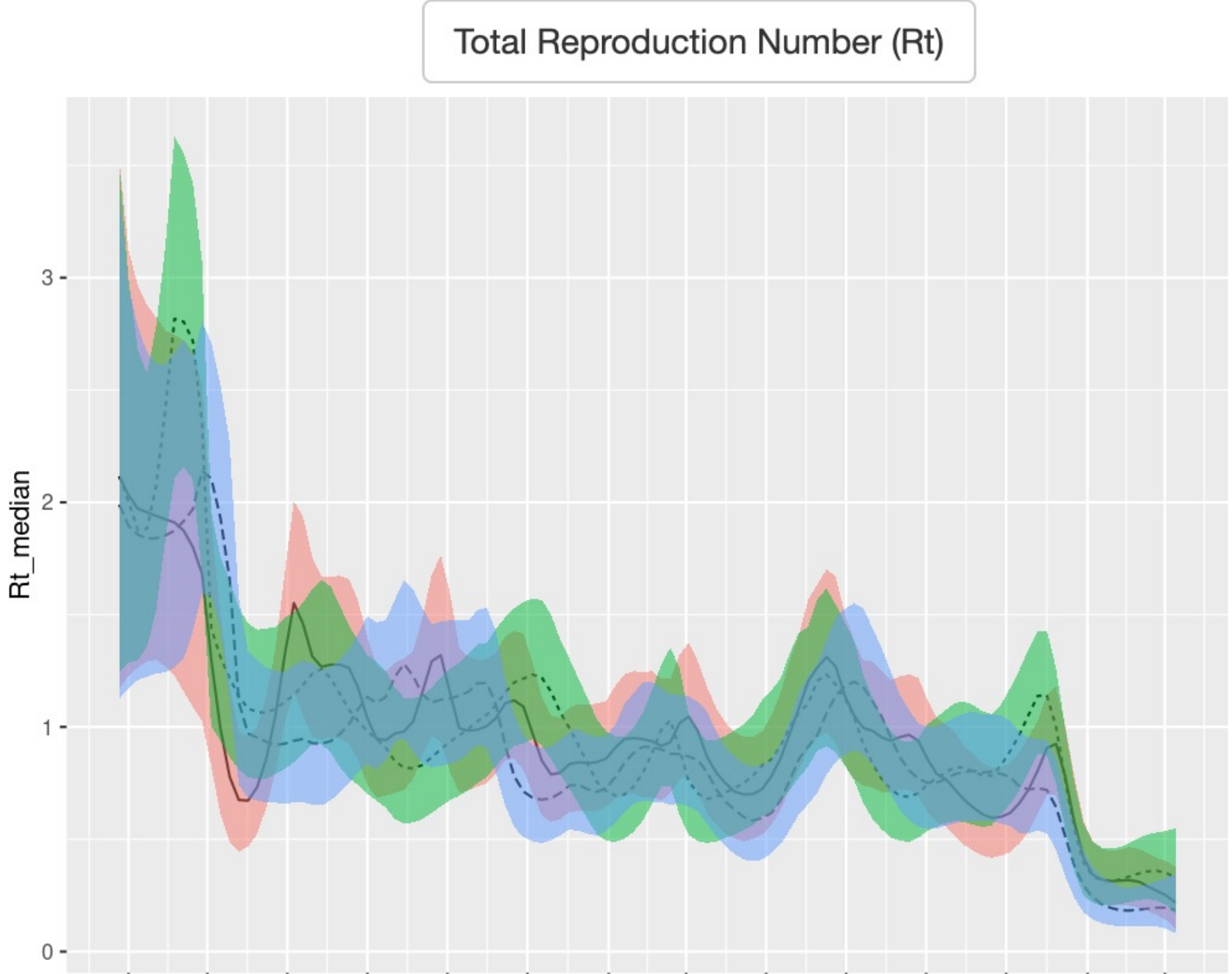


# Reproduction Number ( $R_t$ )



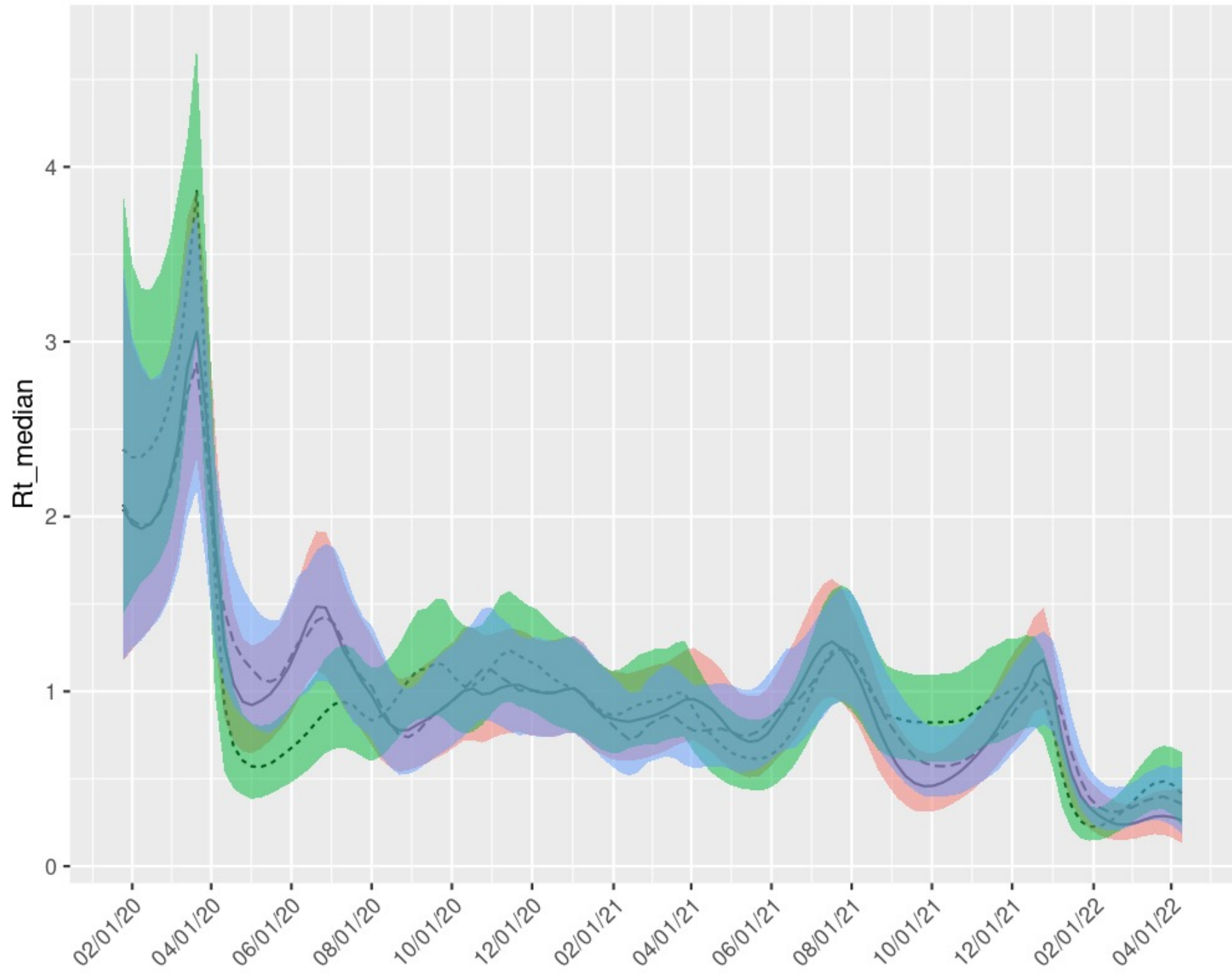
STUSPS

- DC
- MD
- VA



STUSPS

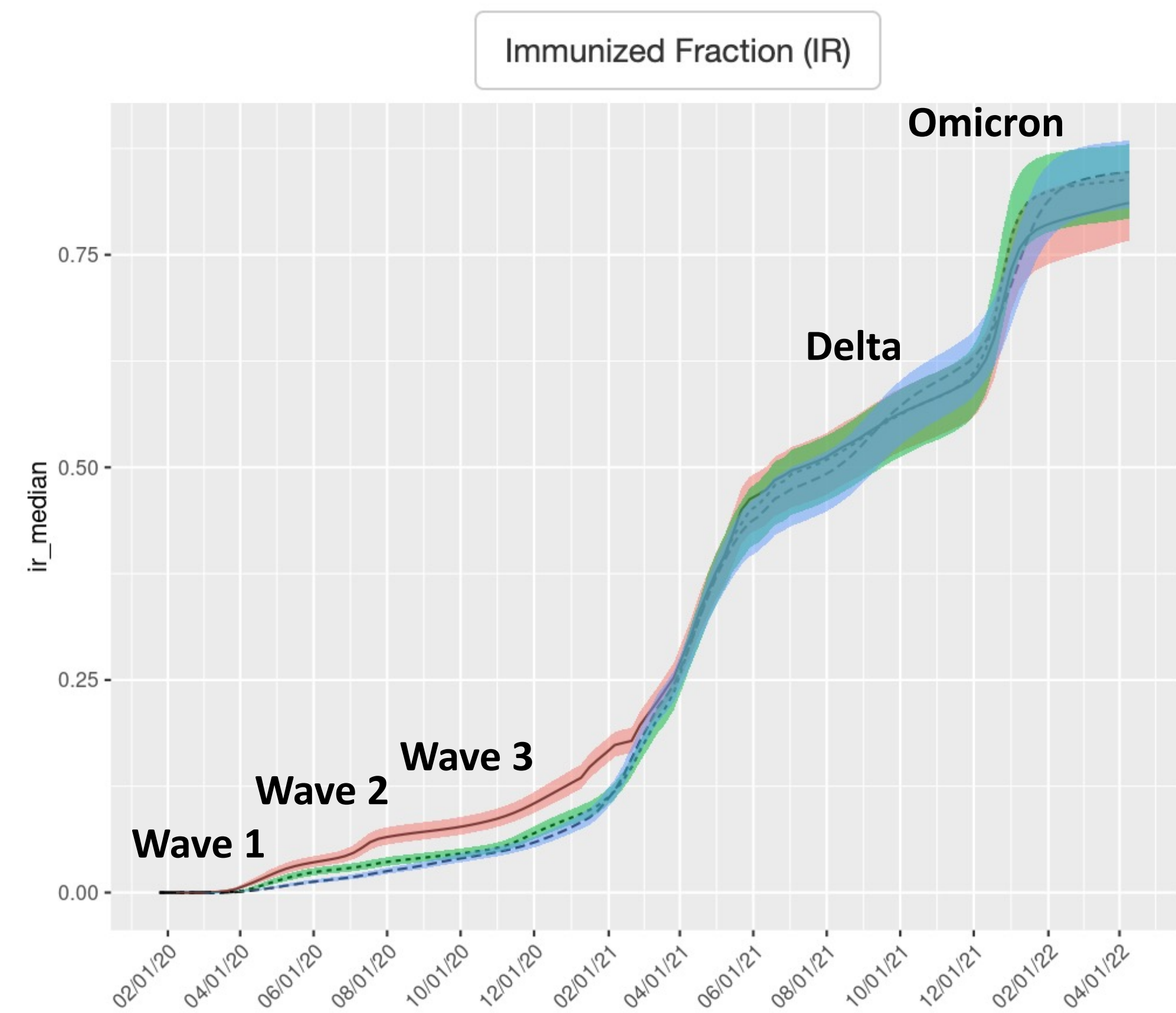
- AK
- CA
- SD



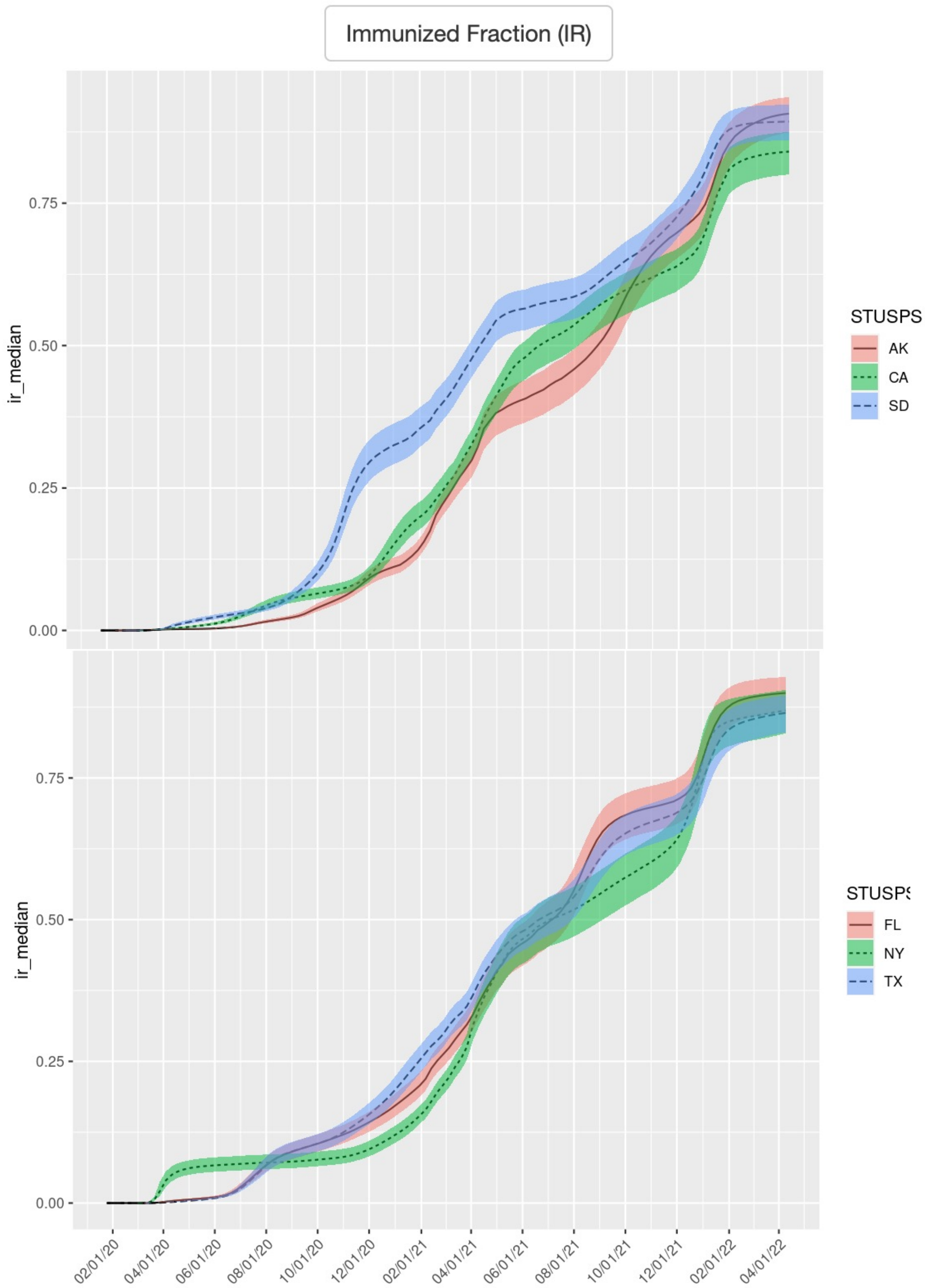
STUSPS

- FL
- NY
- TX

# Fraction infected and/or vaccinated

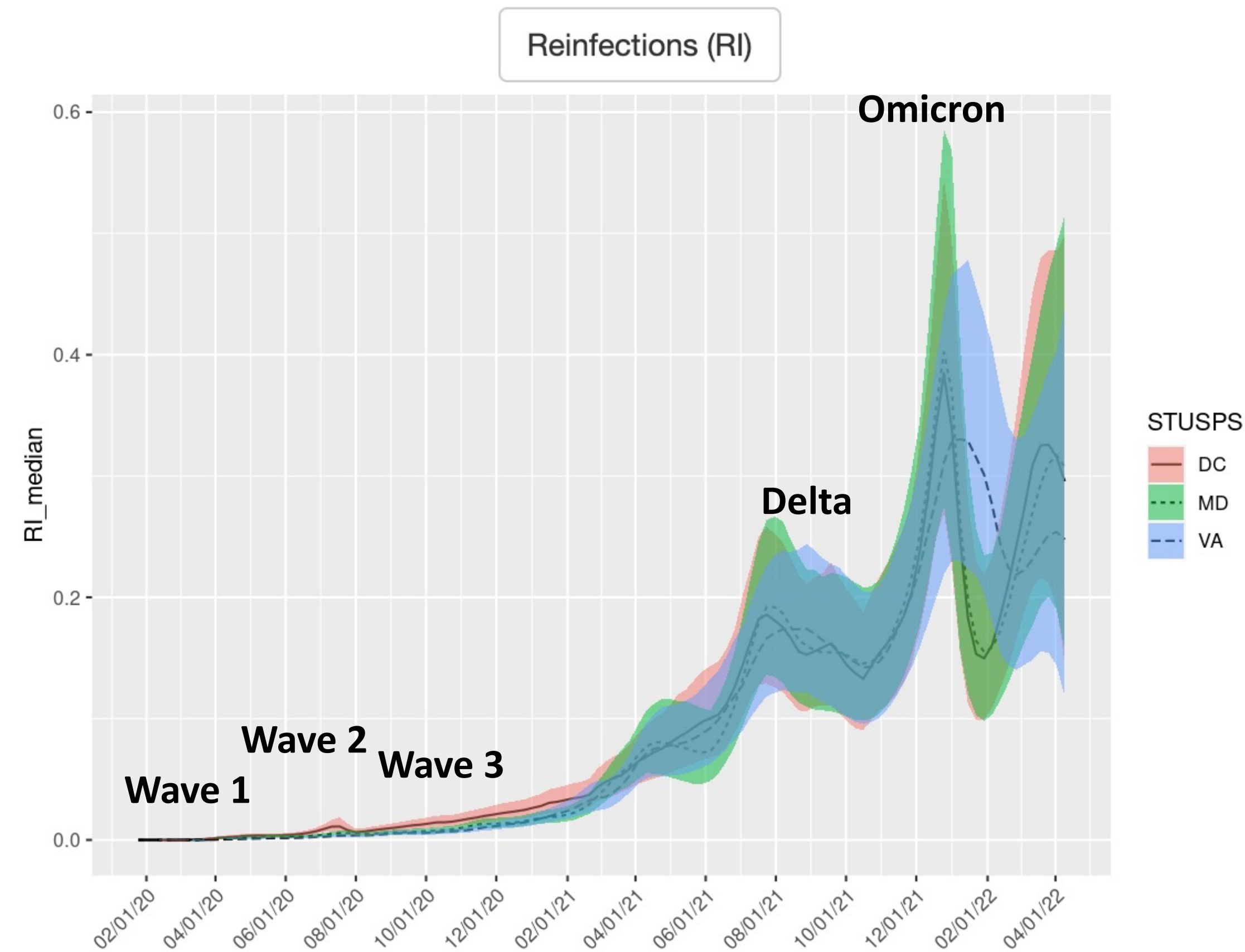


\*Seropositivity priors extend from May 9 - June 25, 2020

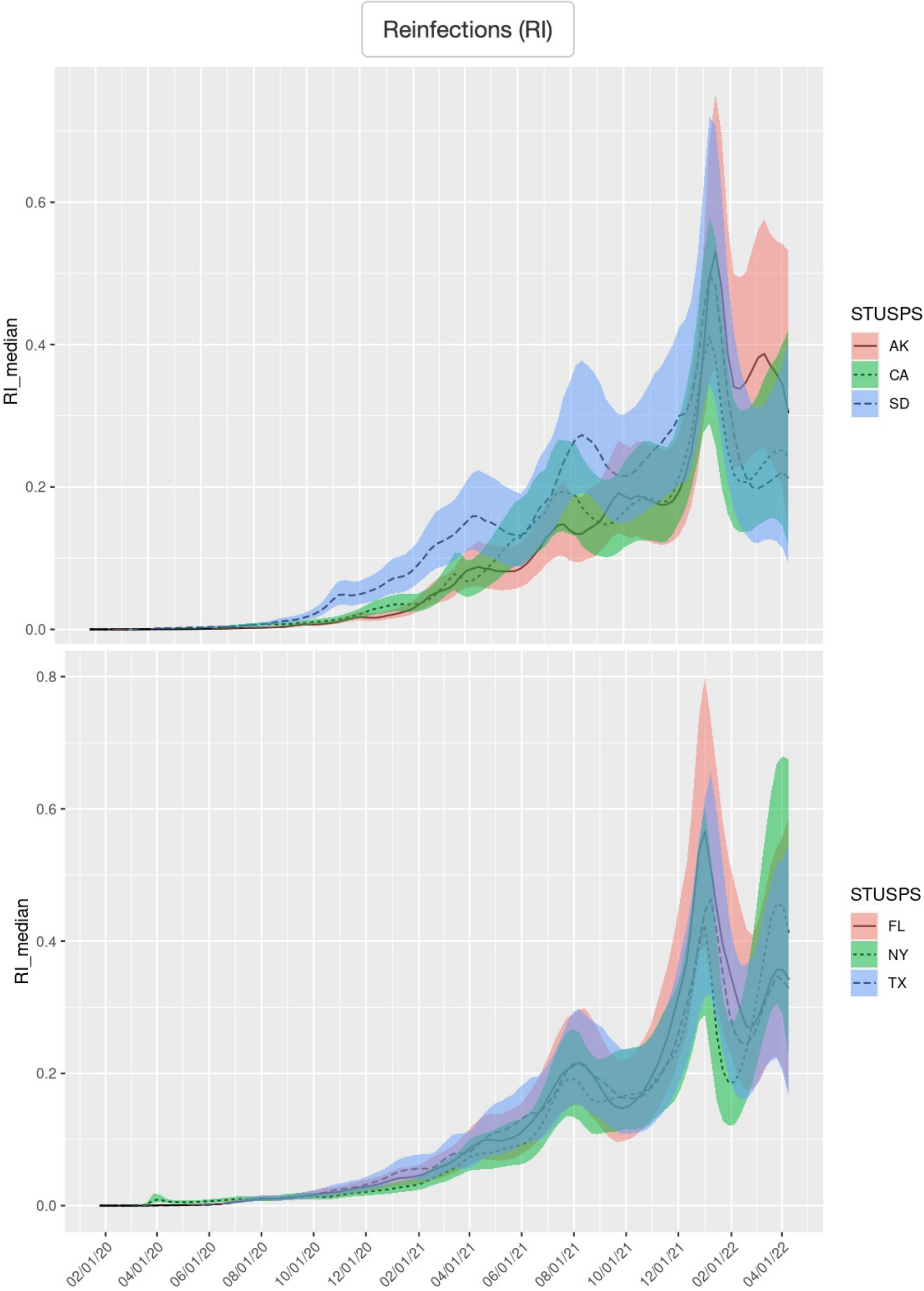




# Fraction of “immunized” population infected ( $RI_t$ )

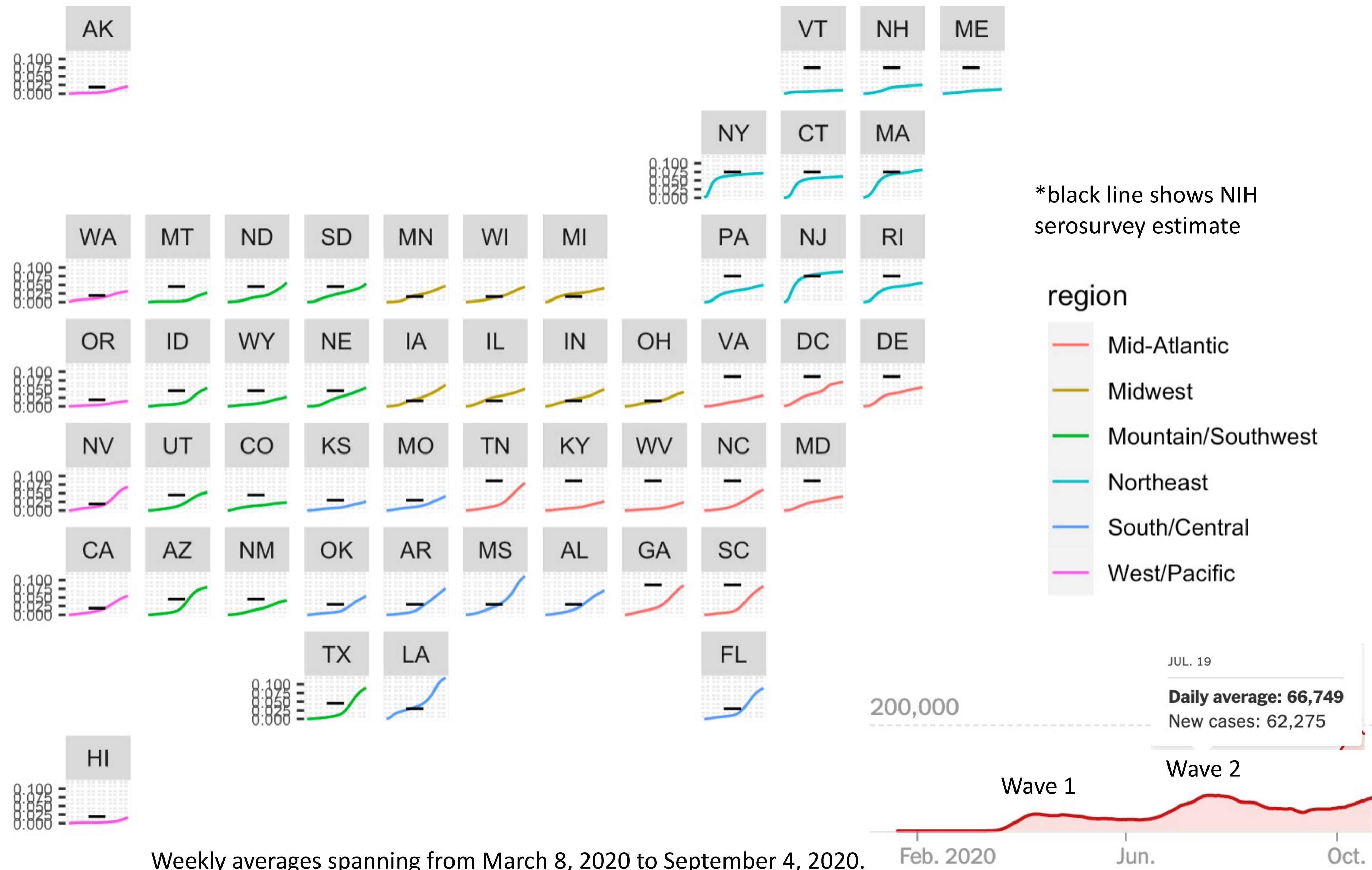


\*Seropositivity priors extend from May 9 - June 25, 2020



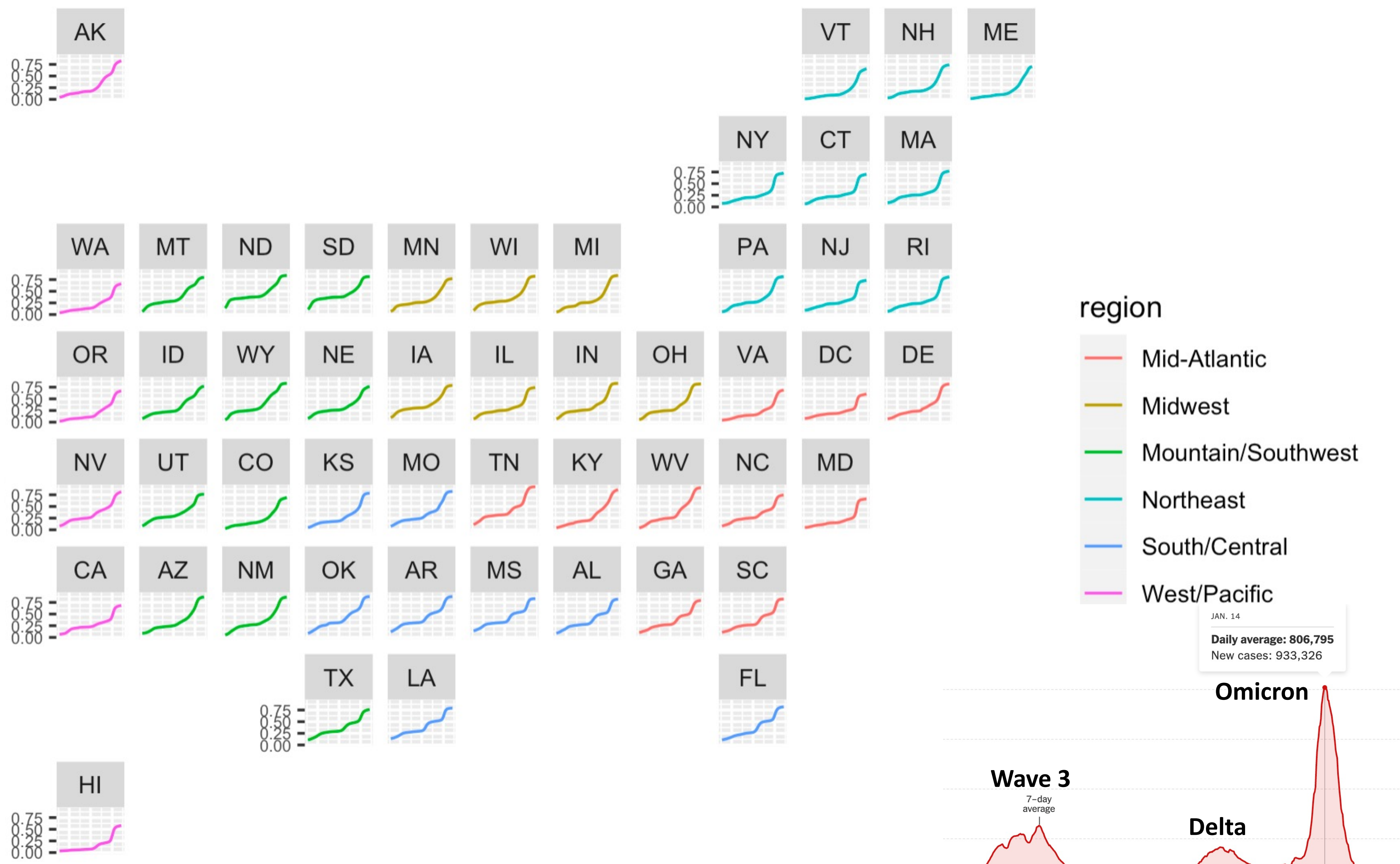


median predicted total infected fraction during Waves 1 and 2

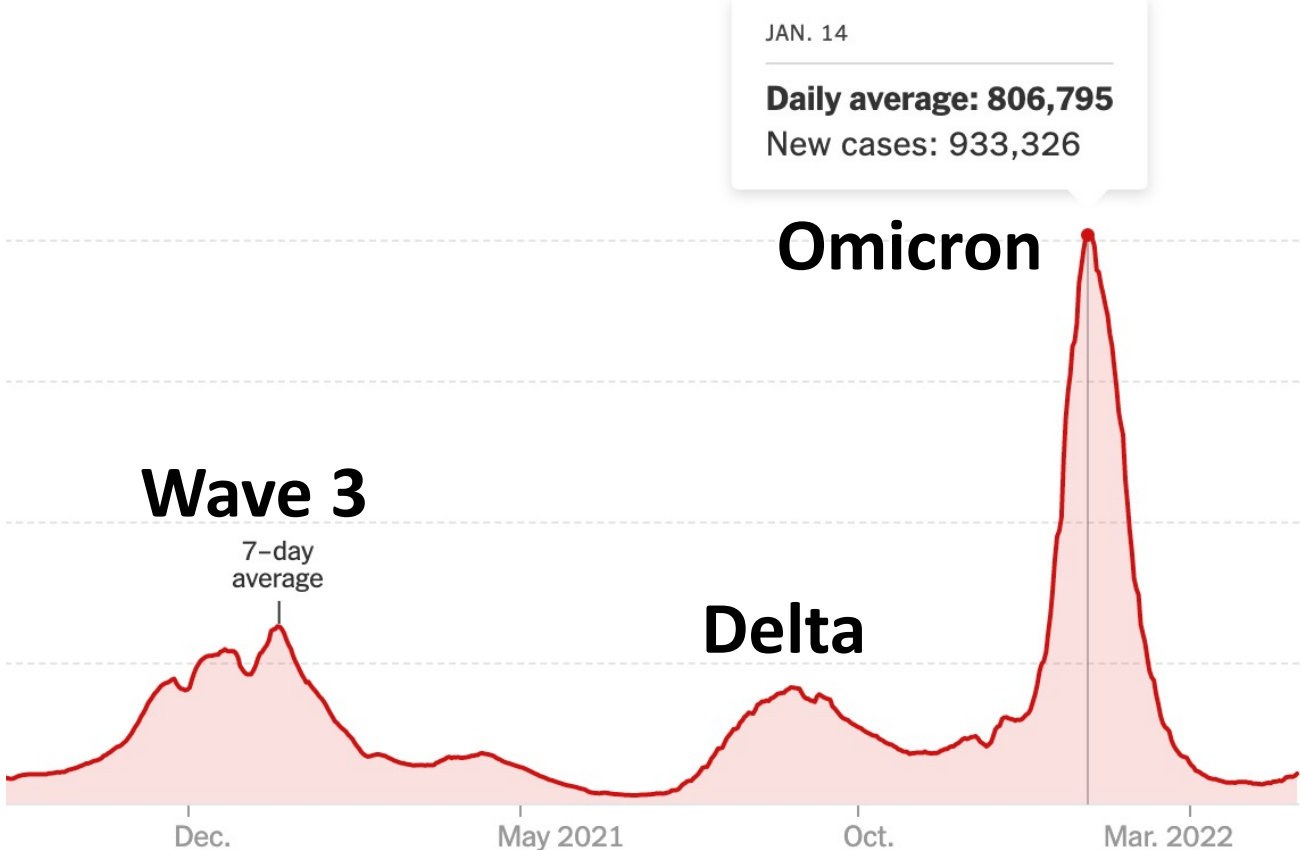




median predicted total infected fraction during Waves 3, Delta, and Omicron



Weekly averages spanning from October 4, 2020 to March 20, 2022.



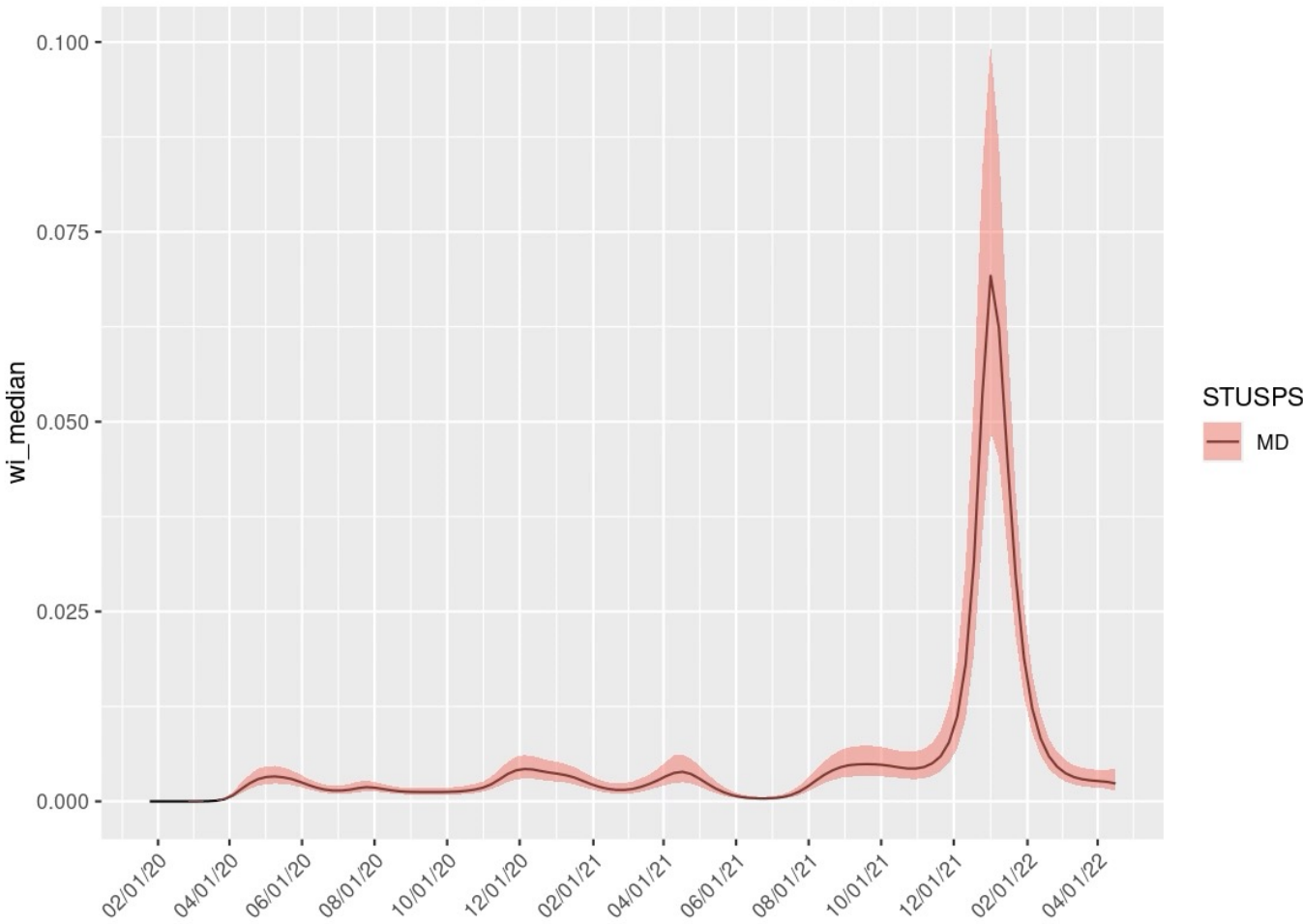
Select State or States

MD

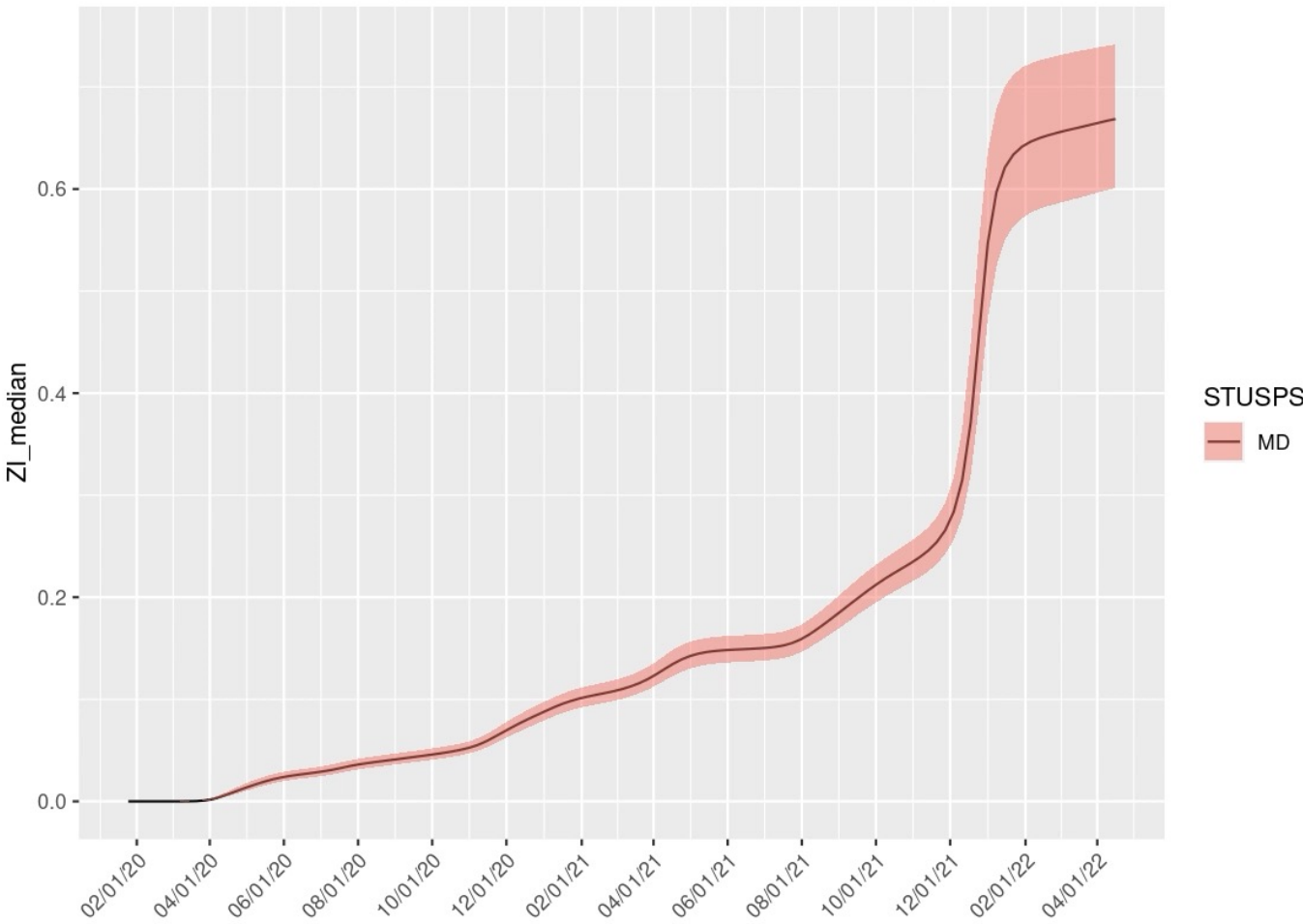
Displaying data up to and including the week of Apr-11 to Apr-17, 2022

Model Predictions

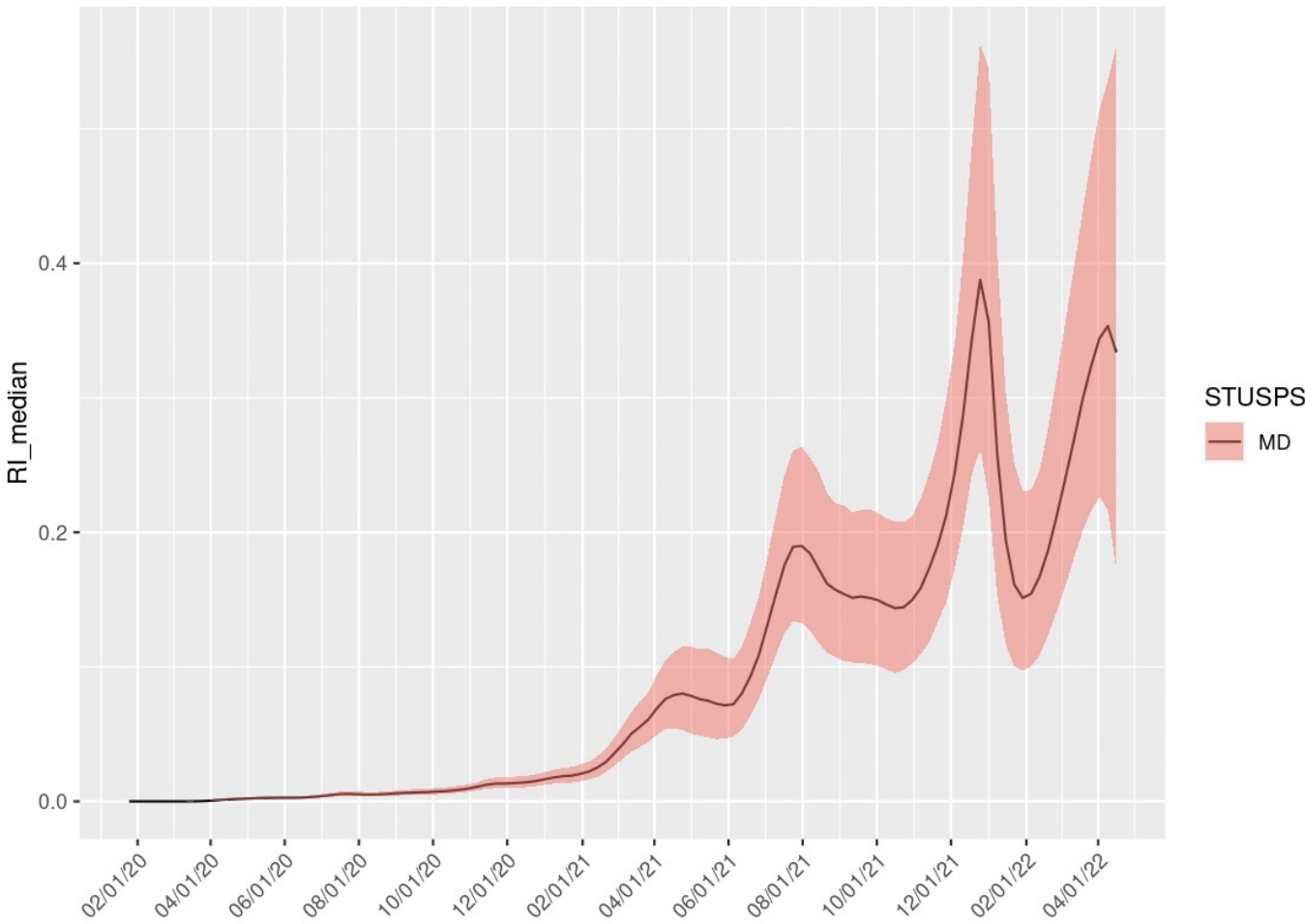
Infected Fraction (WI)



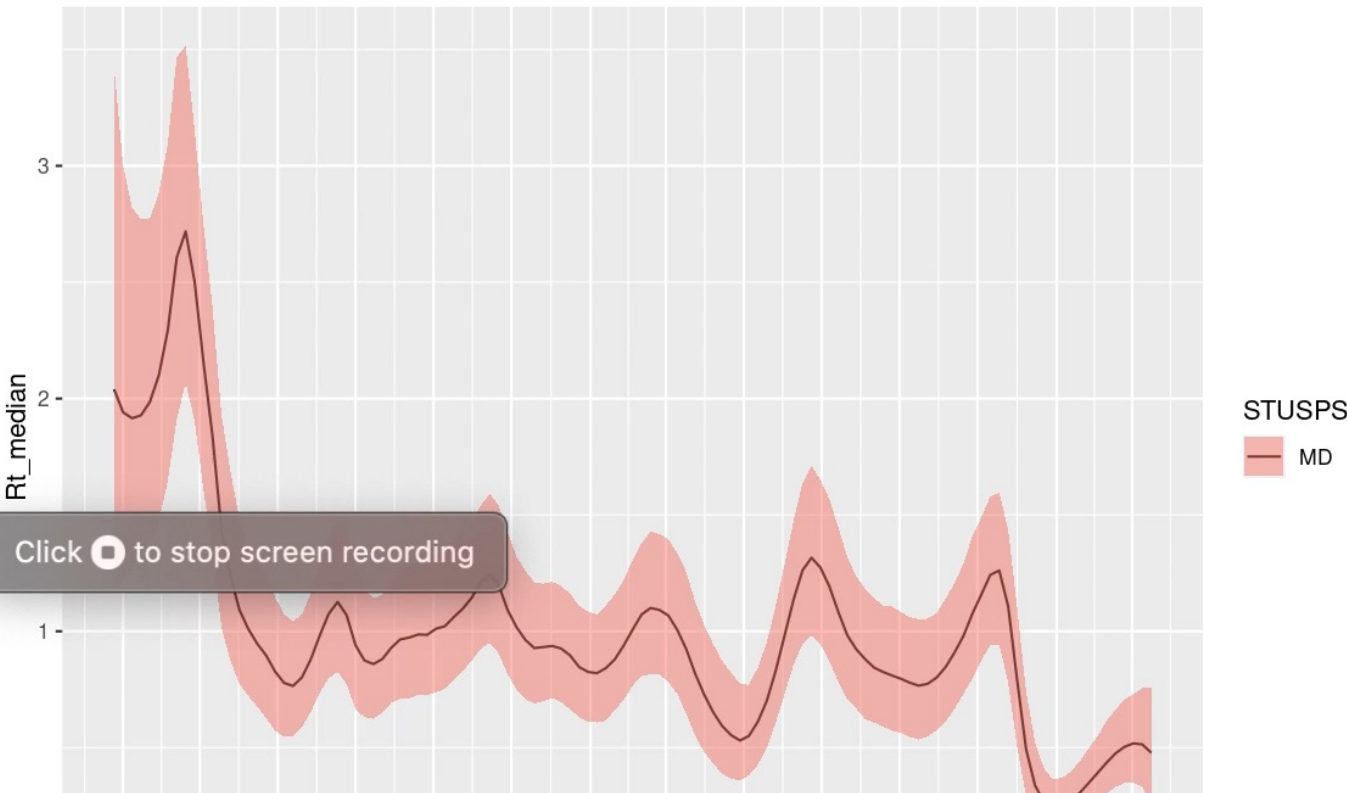
Cumulative Infected Fraction (ZI)



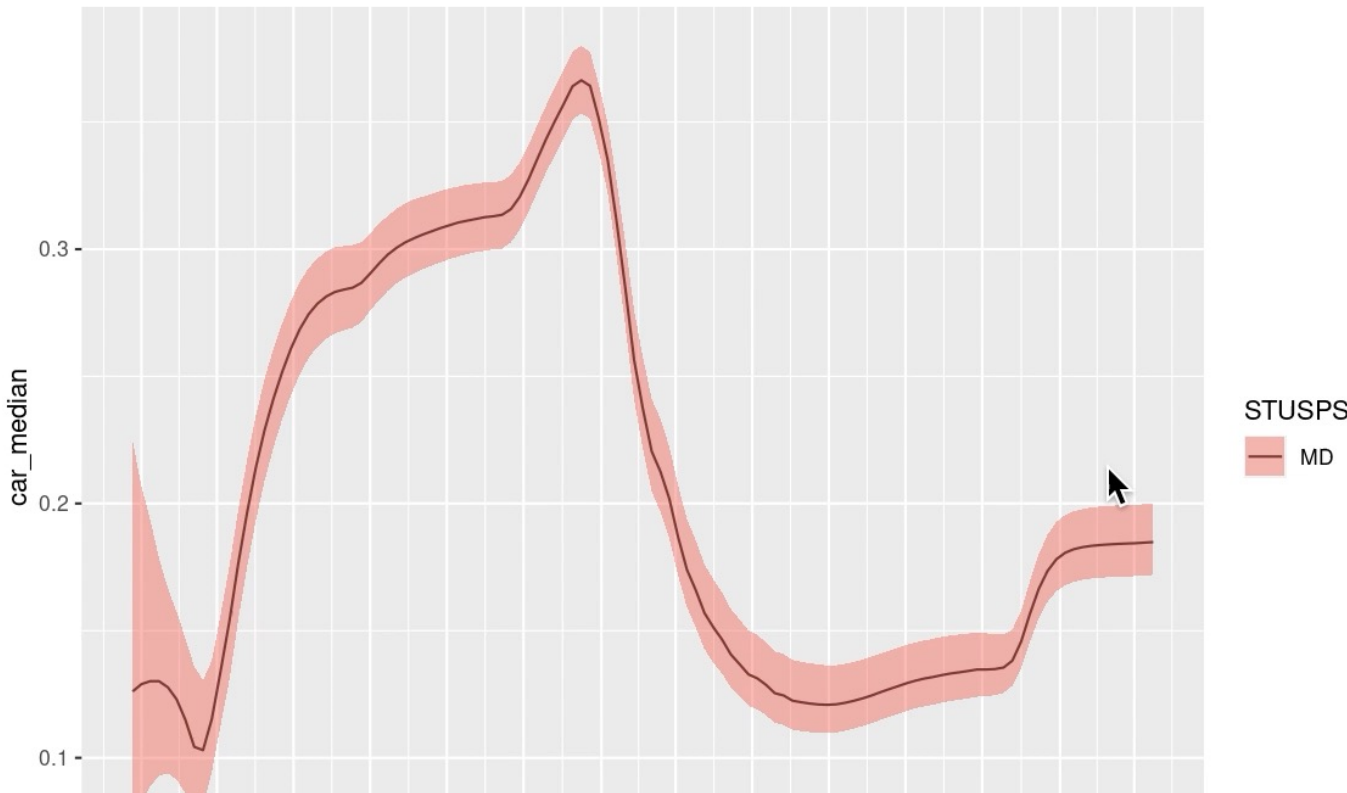
Reinfections (RI)



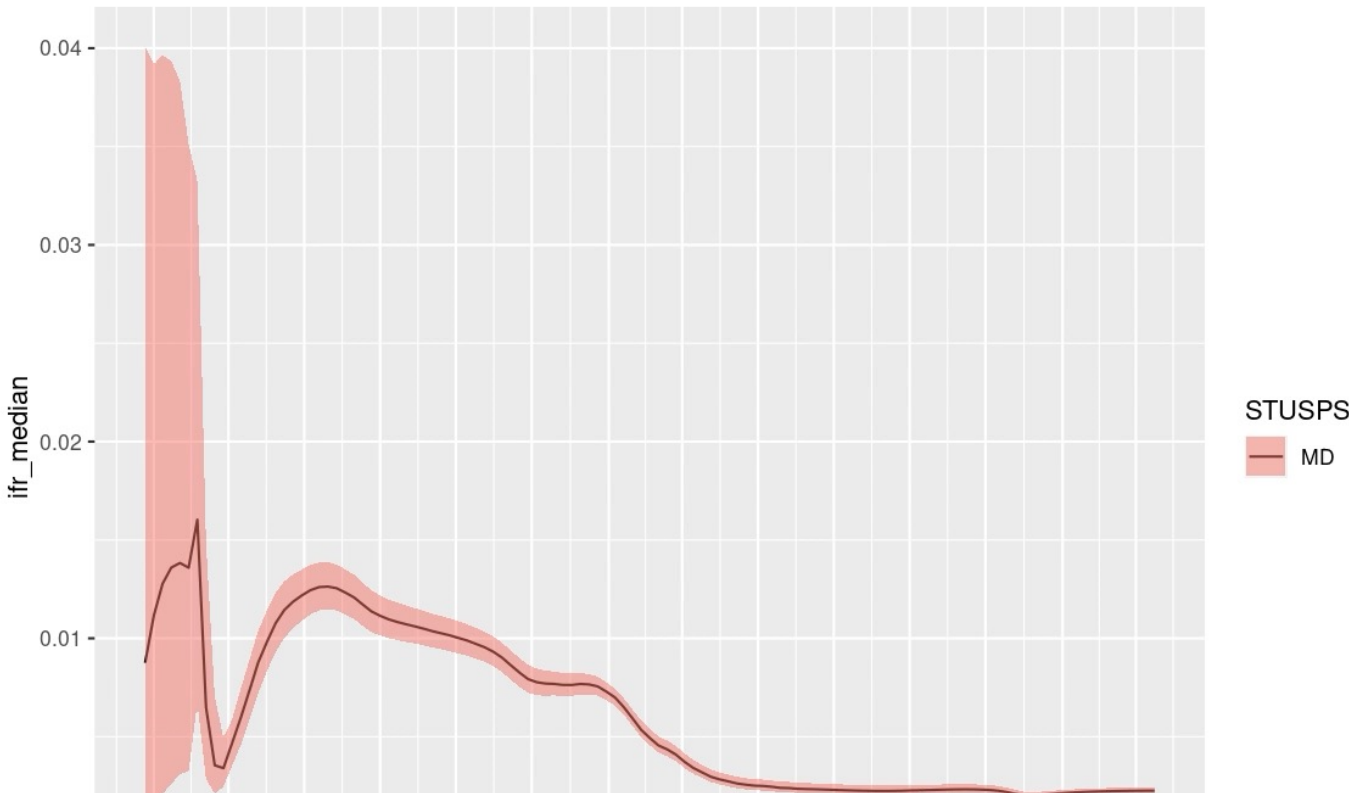
Total Reproduction Number (Rt)



Case Ascertainment Ratio (CAR)



Infectivity Fatality Ratio (IFR)





<https://www.medrxiv.org/content/10.1101/2020.04.29.20083485v1>

<https://github.com/nih-niddk-mbs/covid-sicr>

Slides on [sciencehouse.wordpress.com](https://sciencehouse.wordpress.com)